RESEARCHES IN THE ECONOMICS OF ELECTRIC TRAIN MOVEMENT.

by

John Lundie.

I N D E X.

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SECTION V.

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52 Broadway,

New York City, U. S. A. November 1st 1901.

To the Dean of the Faculty of Science

of The University of Edinburgh.

Dear Sir;-

In accordance with the University regulations governing Graduation in Engineering, the writer respectfully presents the accompanying Thesis, in offering himself as a candidate for the degree of Doctor of Science in Engineering, having graduated as Bachelor of Science in 1880. As required, he hereby declares the Thesis to be a record of original research undertaken by himself.

The Thesis is entitled " Researches in the economics of Electric Train Movement ". The writer does not present it as a complete discussion of the economics of electric railway operation, which, in the heavy traction field, is now little more than in the development stage. He hopes however that the results of his researches on the subject, as presented, may commend themselves as of sufficient value in advancement of the art, to warrant the recommendation of the Faculty of Science for the recognition by his Alma Mater which he seeks.

Respectfully submitted,

RESEARCHES IN TNE ECONOMICS OF ELECTRIC TRAIN MOVEMENT.

I. THE GENERAL PROBLEM OF TRAIN MOVEMENT.

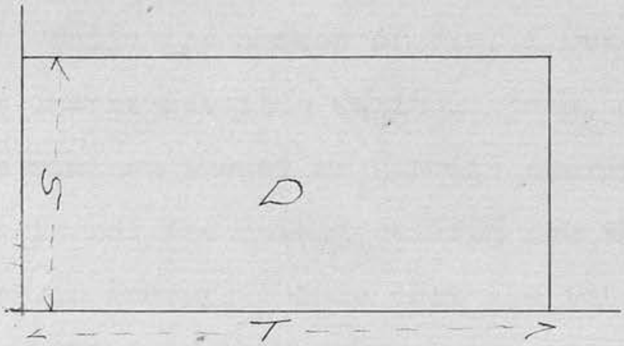
PRELIMINARY.

The problem of economically passing over a given distance in a given time as presented in railway operation with short distances between stations and comparatively high speeds, presents studies in the kinematics and kinetics of the subject which only recently have received the attention due them. This is the class of service to which, in the present state of the art, so called Electric Traction most commends itself, and is that to which the discussions in this thesis are principally applicable.

The general problem is to pass over a distance D, in a time T. The average speed is of course proportional to $\frac{D}{T}$, but the possible variations in speed in performing the movement are infinite.

To illustrate:- Suppose first, the hypothetical case of accelerating a mass M instantaneously on a straight, level track to a speed S; continuing at that speed for a time T and then stopping instantly as represented by Fig. 1. The area enclosed will be a measure of the distance D.

Fig. 1.



Next; suppose the case of uniformly accelerating the mass M during the same time T to a speed $2S$ and again stopping instantly, as represented by Fig. 2.

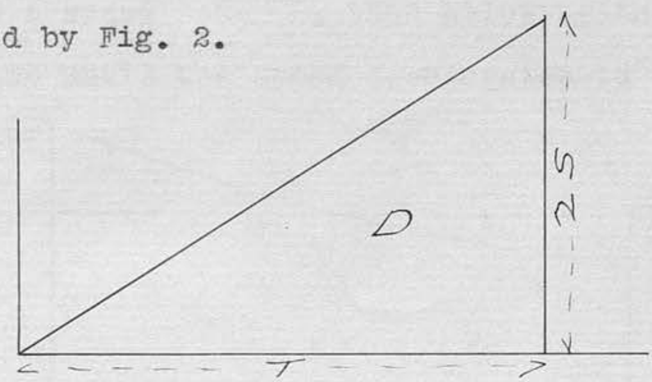


Fig 2

In both cases the same distance is covered in the same time. In both cases approximately the same energy — E — is expended against so called train resistance (see study on the subject). In the first case however, the kinetic energy, (translative and rotative) imparted to the mass is measured by CS^2 whereas in the second case, the kinetic energy is measured by $4CS^2$. In both cases the kinetic energy is assumed to be dissipated in stopping the mass.

It is evident, that hypothetically, by varying the acceleration, any maximum speed might be attained in making this movement and any amount of energy expended in so doing.

While the method of Fig. 1 makes the movement with the lowest possible maximum speed, and consequently with the minimum amount of kinetic energy imparted to the mass, it is not the method calling for the minimum expenditure of energy. Note that the total energy input is that dissipated in stopping CS^2, plus that expended in overcoming train resistance E, or a total of $CS^2 + E$

Now suppose the case of accelerating instantaneously to a speed S_1, then allowing the train to drift for a time until the speed decelerates to S_2 (Fig. 3)

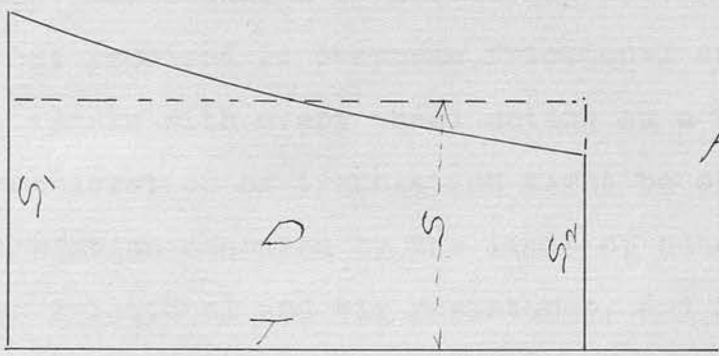


Fig 3.

(the initial speed being such that the distance D will be covered) and instantaneously stopping as before. The energy input here is represented by that dissipated in stopping CS_2^2, plus that expended in overcoming train resistance $CS_1^2 - CS_2^2 = E$, or a total of $CS_2^2 + E$. This is less than the amount of energy required by the method of Fig. 2 by $CS^2 - CS_2^2$, or the difference of the amounts dissipated in stopping.

The nearer the movement approximates the method of Fig. 3, the less the amount of energy required to move the mass M, over the distance D in the time T.

The methods of movement represented by Figures 1, 2, and 3, are not of course practicable methods of train movement, but serve to indicate the general direction in which economy is to be sought.

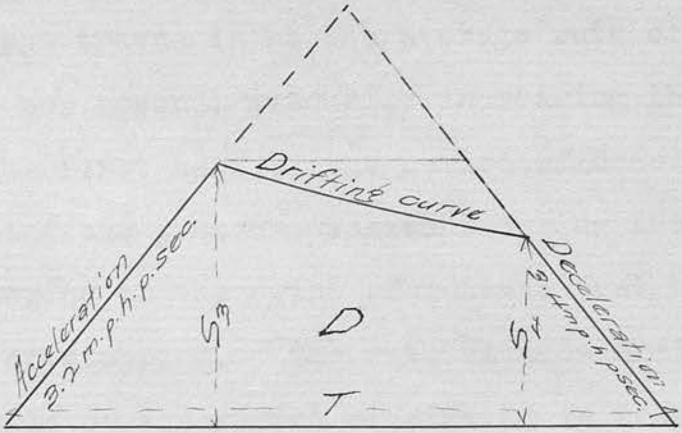
Both the acceleration and deceleration of a train are limited by the adhesion between the driving or braking wheels and the rails. Thus with every wheel acting as a driver, a possible acceleration of translation might be obtained due to the traction measured by the limit of adhesion, less that required to accelerate motion of rotation, and less that required to overcome frictional and air resistance. Again with every wheel acting as a brake, a possible deceleration of translation might be obtained due to the retardation measured by the limit of adhesion, plus that due to frictional and air resistance, and less that required to decelerate motion of rotation.

The co-efficient of adhesion between steel tires and steel rails without sanding, is fairly well defined as from 0.15 to 0.17; thus the maximum acceleration and deceleration would be about as follows:

Maximum acceleration	3.2 miles per hour per second.
Maximum deceleration	3.4 miles per hour per second.

The following method of performing our proposed movement thus presents the nearest possible approximation (without increasing natural adhesion) to the method of Fig. 3 shown to be that requiring the minimum amount of energy applied at the rims of the wheels to move the mass M over the distance D in the time T.

Fig. 4.



The area of the triangle shown in Figure 4, would be a measure of the maximum distance capable of being covered in a time T under the conditions of adhesion assumed.

It might reasonably be noted here, that sanding the rails to increase adhesion can only be considered as an emergency expedient, and ought not to be ordinary working practice in such railway work as we are considering.

The method of Fig. 4, while a possible method of operation presents serious practical objections. Service braking operation, calculated for decelerating a known train mass from a definite maximum speed within a given time, does not permit of immediately applying full brake pressure, for many well known reasons. Thus, the braking curve of a train in service operation would not be a straight line as shown in Fig. 4.

Rates of braking (both emergency and service) with distributed air brake equipment, are so well standardized that no comment need be made on why such rates prevail on well operated railway trains. Approved service braking on suburban trains is at the average rate of 1.5 miles per hour per second; gradually increasing the brake pressure during the first half of the period of deceleration, and maintaining the pressure constant during the second half of the period to the point of release just before the instant of stopping. The rate of deceleration during the second half of the period of braking is something over two miles per hour per second.

The relation between the speed at which brakes are applied and the time and distance of making an average service stop is- Braking distance in feet

$$= \frac{5}{8} \times \text{Velocity in feet per second} \times \text{Time in seconds.}$$

Essentially constant speed electric motors at line voltage, whether direct current shunt wound motors or induction motors, must be brought up to full speed by rheostatic control except as such may be modified by series-parallel or cascade operation; whereas a direct current series wound motor by virtue of its speed characteristic, gains in speed after leaving the rheostat while operating at high efficiency. In such railway work as is under consideration in this thesis, where by far the greater portion of the energy applied is used in overcoming inertia in starting, the selection of the series wound motor, in the existing condition of the art, offers advantages in flexibility of movement and economy, which in the writer's judgment are beyond question.

The general speed characteristic of a train equipped with direct current series wound motors is such that if S_r be the train speed attained by constant torque acceleration on the rheostat in time T_r, then after the elapse of time $2T_r$, the train will have attained a speed of about $\frac{10}{7} S_r$. This speed curve up to this point, is similar to the curve of standard braking; the distance covered in feet being five eighths of the velocity attained in feet per second, multiplied by the seconds elapsed from starting.

Fig. 5.

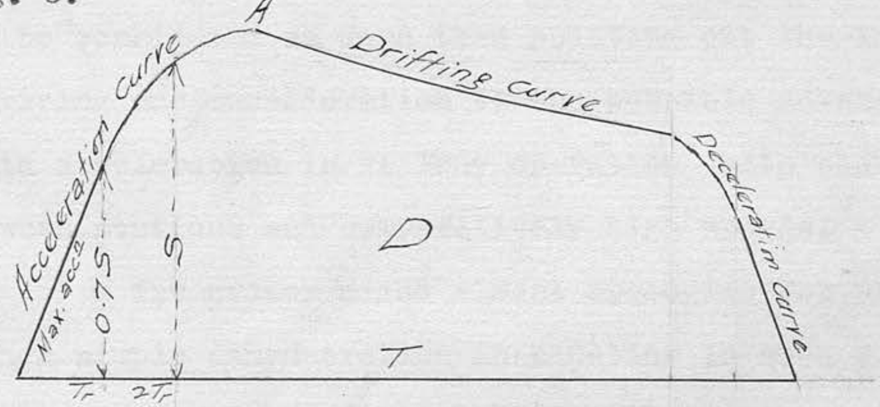


Fig. 5 shows the method of making, under the practical working electric railway conditions indicated, the movement of the mass M over the distance D in the time T, with approximately a minimum expenditure of energy, being as near an approach as possible to the condition of the hypothetical method of Fig. 3.

The position of point A at which power is cut off and drifting begins, would depend on the speed attained and the efficiency between line and rail at which the work of acceleration up to that point had been performed (attaining a lower or a higher speed before leaving the rheostat) in determining the exact condition for minimum expenditure of energy for the movement required.

In an actual railway problem, the distances between stations vary considerably, as do also the conditions of gradients, curvature. block regulations, etc., while the same train equipment must be flexible enough to perform all movements. Again, the power, increased weight and cost of the electrical train equipment required to produce a rapid acceleration, affect the question of final economy. Thus the foregoing discussion, <u>per se</u>, must not be considered as more than pointing out the importance of giving due consideration to the possible advantages of rapid acceleration in railway operation with short distances between stations and comparatively high speeds.

The writer might almost apologize for presenting such a simple demonstration in kinetics in such a thesis as this, but for the fact that he is credited with being the first engineer in electric railway work to point out and demonstrate the value of the principle of rapid accelleration now made such a feature of, in the many distributed motor systems of operation (so called Multiple

Unit Systems) at present in the railway market.

During 1897, the writer was engaged as consulting engineer to the Illinois Central Railway Company, in making an investigation as to the advisability of installing an electric equipment on the surburban trains of that Company. The Company made the sine qua non a very high schedule speed, and the writer very quickly found that such a speed as was asked for was an impossibility unless by virtue of much more rapid acceleration than could be obtained by electric locomotives corresponding to the steam locomotives which were performing the work, because of lack of sufficient adhesion for traction. With the assumption of rapid acceleration, its economy became apparent by the demonstration which precedes. The writer then presented the request to the Electric Companies interested in securing a desirable contract, for the equipment of a trial car with motors of such power as would accelerate it to a speed of forty miles per hour in twenty seconds. The proposition was laughed at and remained tabled for months. The commercial manager of the General Electric Company finally agreed to have a small car equipped just as desired, and the writer went to the Company's Works at Schenectady August 1st, 1897 for the purpose of having the test carried out. On August 4th, the first run was made without any "hitch", and the ease and economy (per unit of mass) of the performance demonstrated.

Exhibit A is an exact copy of the record of the performance of this car. Two months later- October 1897-

the General Electric Company made an elaborate demonstration of the possibilities of rapid acceleration when Lord Kelvin was visiting the Schenectady Works.

TRAIN MOVEMENT DIAGRAM.

In order to compare the possibilities of movement over varying distances in varying times, and with adhesion of various proportions of the weight of a train utilized for traction, the writer designed a movement diagram on the following basis:

First- Assuming acceleration on the speed characteristic described for series wound motor equipment, so that the total time of acceleration is twice that occupied in uniform acceleration on the rheostat, and the speed attained 10/7 of that on leaving the rheostat.

Second- Assuming the train runs at that uniform speed until the brakes are applied.

Third- Assuming braking on the service curve before described at the rate of 1.5 miles per hour per second from the point of application of brakes.

The acceleration and braking curves are similar in horizontal projection, although the braking curve is assumed at the constant rate named, while the acceleration curve varies in its time element depending on the adhesion utilized.

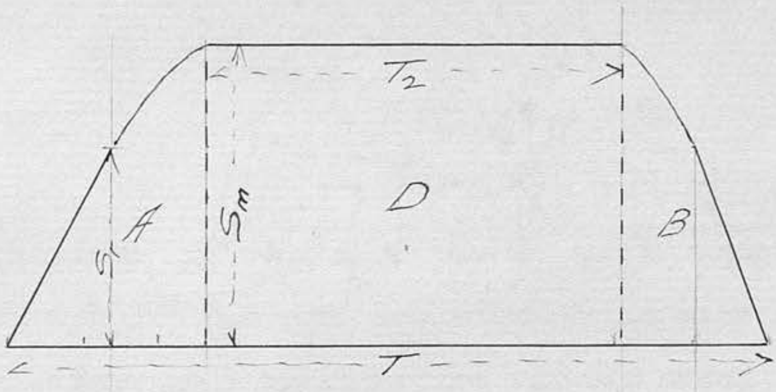
The assumption of a uniform full speed between acceleration and deceleration, is a perfectly fair one, as with longer or shorter station distances in actual operation, acceleration would simply continue beyond the point indicated for such a time that in conjunction with the drifting period what might be termed "average full speed" would be maintained the same.

As previously noted, the work to be done during the acceleration of a train on a straight level track, consists of overcoming frictional and air resistance (train resistance), overcoming the inertia of translation of the whole mass, and overcoming the inertia of rotation of the revolving parts. A simple allowance may be made for the inertia of the rotating parts by adding an equivalent to the mass of the train in calculating energy of translation. For American trains electrically equipped with distributed motors, this equivalent may be taken at not to exceed ten per cent of the mass to be moved, so that in calculations pertaining to the energy required for any train movement, a virtual mass may be assumed of 1.1 x M, and this virtual mass treated as if it simply had inertia of translation.

So far as the possibilities of acceleration are concerned as dependent upon the proportion of weight on the driving wheels, the limiting co-efficient of adhesion may be assumed to be 0.15 and the total mass treated as subject only to inertia of translation and frictional and air resistance; the excess of adhesion over this assumed limit being allowed to take care of the inertia of the rotating parts.

The following is a description of the construction of the Train Movement Diagram: Exhibit B. The abscissae of the diagram are the combined distances $A+B = D_1$ of acceleration and deceleration in feet. The ordinates are speed in miles per hour.

Fig. 6.



Referring to fig. 6, let

D = average distance between stations in feet

S_m = average maximum speed in miles per hour

$V_m = S_m \times 1.467$ = maximum velocity in feet per second

$D_1 = A + B$ = distance of accelerating and decelerating in feet

$T_1 = T - T_2$ = combined time of accelerating and decelerating in seconds.

<u>First, determining relation between time and distance for accelerating and decelerating. -Blue polar lines-</u>

$$D_1 = 0.625\, V_m \times T_1 \qquad \text{as previously shown}$$

$$= 0.917\, S_m \times T_1 \text{ ------------------ } (1)$$

The equation relating D_1 and S_m for a given T_1, is thus a straight line through the origin.

Second: Determining relation between average speed and maximum speed. -~~Black~~ Red lines-

The lines expressing this relation vary in inclination for each D ; all other lines remain fixed for any value of D.

$$D_1 = (T.V_m - D)\frac{0.625}{1-0.625}$$

$$0.6D_1 = TV_m - D$$

$$V_m = \frac{0.6D_1}{T} + \frac{D}{T}$$

$$S_m = \frac{0.6\,D_1}{1.467.T} + \frac{D}{1.467\,T} \text{ ------ } (2)$$

For a given D and a given T, the equation relating D_1 and S_m is thus a straight line with an intercept on the axis of Y equal to the average speed $\frac{D}{1.467\,T}$ When $D = D_1$, or when the whole distance is occupied in accelerating and decelerating, then the ordinate S with an abscissa equal to the whole distance $= 1.6\frac{D}{1.467\,T}$ or 1.6 times the average speed.

This relation offers an easy means of changing the inclination of the lines relating average speed and maximum speed, the ordinate at $D_1 = D$, for varying lengths of total run D being a constant.

Third: Acceleration as dependent on percentage of weight utilized for traction, -~~red~~ black lines-.

With any reasonably rapid acceleration, the force required to overcome the inertia of the mass moved would be great in proportion to that required to overcome frictional and air resistance, besides the latter is a somewhat uncertain quantity in starting, so that it may readily be given an average value during the period of all ordinary acceleration on the rheostat without introducing an appreciable error. This resistance, the writer after due consideration has assumed at an average of 11.4 pounds per ton (2000 pounds).

Let now x = percentage of weight of train on driving wheels,

W = weight in tons (2000 pounds)

P = pounds traction.

Assuming as before explained, adhesion 0.15 as applied to motion of translation and train resistance, then,

Pounds traction per ton available for acceleration of translation: $= \frac{P}{W} = \frac{x}{100} \times 0.15 \times 2000$

$$= 3x$$

$F = aM + f$ - From principles of kinetics.

$$P = \frac{S \times 1.467}{t} \times \frac{W \times 2000}{32.2} + W \times 11.4$$

$$3x = \frac{P}{W} = \frac{S}{t} \times 91.1 + 11.4 \text{ - - - - - - - - - - - } (a)$$

$$x = \frac{S}{t} \times 30.4 + 3.8$$

The above S represents speed on leaving the rheostat and equals $0.7 \times S_m$; while t represents time on the rheostat and equals one half of the time t_m attaining the speed S_m.

Substituting:

$$x = \frac{S_m \times 0.7 \times 30.4 \times 2}{t_m} + 3.8$$

$$x = \frac{S_m}{t_m} \times 42.6 + 3.8$$

Acceleration in miles per hour per second to speed S_m =

$$A = \frac{S_m}{t_m} = \frac{x - 3.8}{42.6} \text{ --------- } (3)$$

The rate of braking being 1.5 miles per hour per second, we have

Total time of acceleration and braking, (Fig 6) =

$$T_1 = \frac{S_m}{A} + \frac{2}{3} S_m$$
$$= S_m \left(\frac{1}{A} + \frac{2}{3} \right)$$

Now from equation (1)

$$D_1 = 0.917 \, S_m T_1$$
$$= 0.917 \, S_m^2 \left(\frac{1}{A} + \frac{2}{3} \right)$$

$$S_m = \sqrt{\frac{D_1}{0.61 + \frac{0.917}{A}}}$$

substituting for A

$$S_m = \sqrt{\frac{D_1}{0.61 + \frac{39}{x - 3.8}}} \text{ --------- } (4)$$

For a given percentage of the weight of a train on the drivers, the equation relating S_m and D_1 is thus a parabola.

The train movement diagram exhibit B, being mathematically correct on the basis of the assumptions made; from it can be quickly determined under any stated conditions the possibilities or limitations of train movement.

On a large wall diagram in the writer's office, the variable lines showing the relation between average and maximum speed, are substituted by silk threads readily set for any distance of run .

II. CHARACTERISTICS OF THE ELECTRIC RAILWAY MOTOR WITH REFERENCE TO TRAIN MOVEMENT.

PRELIMINARY.

With a given potential difference between the terminals of a given direct current series wound electric motor, there exists for any current passing through the motor a corresponding armature speed; the potential difference between terminals always being the sum of the varying counter electro-motive force developed and the drop due to resistance. This relation between current and speed is usally called the speed characteristic of the motor. Exhibit C is a copy of a general characteristic sheet issued by The Westinghouse Electric & Mfg. Co., of a modern 500 volt railway motor, rated at 75 brake horse-power on the basis of the existing American manufacturers' standard method of rating; viz:- A brake output of 75 horse power for one hour, during which time the windings rise in temperature 75 degrees C. the temperature at starting being about that of the normal atmosphere. It will be noted that the current at 500 volts producing the output named is about 130 amperes, and the corresponding armature speed about 500 revolutions per minute.

Exhibit D is a copy of a general characteristic sheet issued by the General Electric Co. of a 500 volt motor, rated at 80 horse power at an input of 140 amperes.

The writer purposely avoids detailed discussion in this thesis of the relations of field and armature windings and of the magnetic circuit to the speed characteristics of the motor, simply considering any given motor as a machine with defined characteristics, through which electric energy is converted into mechanical energy, applicable for purposes of traction.

Were such a series wound railway motor frictionless, its speed would approach infinity as the current approached zero. Again, with increase of current and a consequent high degree of magnetic saturation, the speed approaches a constant. Thus, in an endeavor to obtain a general expression relating the speed and current of motors in preliminary consideration of railway problems, the writer has made a comparison of the speed characteristics of the principal modern railway motors manufactured in the United States, and has found as a result of inspection, that within working limits, a rectangular hyperbola whose asymptotes are the axis of Y, and a line parallel to the axis of X, will co-ordinate current and speed within ordinary limit of error. This tabulation forms Exhibit E, in which A represents the constant product of X and (Y - minus an intercept).

The working limits of a motor require definition. Practically sparkless commutation is usually obtained on a well designed motor with a current exceeding that of rated full load by fifty per cent, and characteristic sheets supplied by manufacturers generally extend to this limit.

The writer's experience has indicated that a fifty per cent overload above standard full load current, is too high for average starting current when operating under the action of a notching rheostatic controller.

In stepping from notch to notch of a controller, the current does not remain actually constant for an instant, being modified by the resistance introduced on each notch, by induction and by the speed characteristic of the motor. The object sought by a controller, is to keep the average current per motor during the cutting out of resistance, as nearly as possible, a constant. Depending on the number of notches, the actual current on each notbh may however run considerably above and below the average. While acceleration on a stepped controller thus consists of a series of impulses, it may, for all practical purposes, be considered as that produced by the traction due to the average current .

This average advisable starting current , so far as the commutation limit of a motor is concerned, the writer considers as fairly well defined by twenty-five per cent excess of rated full load. This may be considered, for purposes of calculation, as the upper working limit of a motor.

The lower working limit of a given motor in a given service, is defined by the current corresponding to the approach to a constant maximum speed under conditions of minimum resistance to motion. This will seldom be less than one fifth of the upper working limit.

The upper working limit of a motor, will serve to indicate a capacity suitable for a given train movement so far as maximum power requirement is concerned. This criterion alone does not however serve in the selection of a motor for a given service. The fact that standard full load is determined by a limiting temperature in the windings of a motor, indicates that the temperature at which a motor may be operated, is just as important a factor in its selection for any specific service as is a satisfactory commutation limit. It becomes necessary throughout the continually varying load, to which a railway motor is subjected, in such service as is under discussion, to determine the conditions under which the motor will reach the maximum temperature allowable, being that which will not cause undue deterioration to the materials or construction of the machine.

In the field and armature windings of the motor under consideration, the fraction of energy lost in heat due to resistance on the passage of any current I, is protional to $\frac{I^2}{I}$ or directly as the current. At the same time, the fraction of energy lost due to core heating, increases with the speed and diminishes with increasing current

The following general statement may be made, as the result of inspection of the efficiency characteristics of the motors referred to in exhibit E;-

<u>The electrical efficiency of any of the motors described, throughout its working range of current, is approximately a constant.</u>

Thus, no matter what the variation in current may be in the operation of a motor, the energy lost in heat is approximately a constant fraction of the input. If now the dissipation of heat from the motor is a function of the temperature difference between that of the motor and its surroundings, then under any variations of continued rapidly applied intermittent load, the average input will determine the temperature at which heat dissipation will equal heat generation.

The reduced core heating on currents under rheostatic control, the better ventilation at higher speeds and the localization of heating all tend to modify, at varying currents, the rate of generation and of dissipation of heat. Again, with series-parallel control, if we consider the average current per motor as the average current per car, divided by the number of motors thereon, we will obtain a figure for average current slightly lower than the true average, seeing the same current passes through two motors when in series position. Under the conditions of railway service treated of, the statement however is justified that, within the range of the working current limits of the motor, the average input per car divided by the number of motors, will give an average current per motor which (independently of such variations as it may be subject to in service operation) will determine the maximum working temperature of the motor windings.

If I_A represent average current in amperes, then under the rigorous application of this principle, if

I represent any current within the working range, and

T represent the percentage of time it is applied during continued intermittent application, then

$$I_A \times 100 = I \times T$$

Thus, a motor intended to take an average of 60 amperes (not necessarialy 60 amperes continuously) with a working commutation limit of 300 amperes, ought to take

300	amperes	20%	of the time
200	"	30%	"
100	"	60%	"

with approximately the same limiting temperature.

In stationary test on this basis, the heating in approaching the commutation limit would be somewhat greater than in service, owing to the modifications in service operation pointed out. This however, can be said in justification of a stationary test- if the temperature requirements are met on stationary test, the motor will be perfectly safe on temperature limitation in service.

Such a series of tests were carried out by the writer in selecting the motors for the service of the Boston Elevated Railway. The results of two tests on one of the motors are shown in Exhibits F and G, illustrating an approximation to the same ultimate maximum temperature under currents widely divergent as to intensity, but averaging the same.

The average current per car divided by the number of motors on the car , will give an average current per motor which will produce a temperature elevation in well designed motors, not exceeding 75°C above the surroundings, when this average current per motor is about 25 per cent of standard rated full load current, or one fifth of the working commutation limit.

For preliminary determination of motor capacity, we have thus the following criteria;

1- Working commutation limit- 25 percent in excess of rated full load current.

2- Thermal capacity limit- an average intermittent current input of 25 per cent of the rated full load current.

In order to reduce losses which would otherwise occur in purely rheostatic speed control, motors are usually installed in pairs so as to permit of starting them connected in series and subsequently throwing them into parallel relation between line and ground, maintaining, as nearly as possible, a constant average current per motor, by virtue of properly graded resistance introduced into the line circuit, and gradually cut out in passing from notch to notch of the controller.

In a given motor operating under constant current by virtue of varying the potential difference between terminals, the speed at any instant is proportional to the counter electromotive force generated therein. Assuming the drop due to resistance of the motor to be 50 volts (which is not far from standard practice) at the upper working current limit, we have in series position of a pair of 500 volt motors without resistance in circuit, a speed represented by

$A(250-50) = 200A$. Again, in parallel position without resistance and with the same current per motor, we have a speed represented by $A(500-50) = 450A$.

Thus, in seeking a uniform acceleration by series-parallel operation of the controller to a speed $450A$, we have at the instant of clear series operation, just before throwing into parallel, a speed of $200A$. The ratio of these speeds is about 1 to 0.45; thus for uniform acceleration on constant current per motor under series-parallel control, the time of remaining in series position is conveniently calculated at 45 per cent of the total time on the controller.

DEVELOPMENT OF GENERAL MOTOR FORMULAE.

In now proceeding to develop some general formulae coordinating the speed and power characteristics of the electric motor, applicable to preliminary calculations in attacking a railway problem, it is assumed;-

1- That the maximum traction available for service operation, is 330 pounds per ton (2000 pounds); that 300 pounds of this traction is available for overcoming inertia of translation and train resistance, 30 pounds being utilized in overcoming inertia of rotation of revolving parts.

2- That the maximum working current limit of the motor will be utilized in accelerating the train during the operation of the controller.

3- That throughout the working current range of a motor operating on line p.d. of 500 volts, the ratio of mechanical energy output at the rail to electrical energy input is 80 per cent.

4- That driving wheels are 33 inches in diameter (now standard in the United States). If for any reason a different diameter is desirable, the gear ratio G in the following formulae may be altered to conform to such a change.

Let

I_r = Maximum working current limit of the motor in amperes per ton of mass moved.

x = Any fraction of the maximum working current of the motor I_r.

I = Current $(x \times I_r)$ per ton at speed S.

F = Pounds traction per ton

P = Pounds 'Train Resistance' per ton as per formula q. v.

D = Fraction of weight of train utilized on driving wheels. Note- x was used for percentage of weight on drivers in a former formula.

V = Velocity in feet per second.

S = Speed of train in miles per hour, at current I

S_r = Speed of train on leaving controller

S_m = Average full speed

N = Number of revolutions per minute of motor armature

T_r = Time on controller in seconds

T_m = Time to average full speed

G = Gear ratio (Number of teeth in gear wheel on axle divided by number of teeth in pinion)

Referring now to exhibit E of the speed characteristics of modern railway motors and to the fact that current and speed are coordinated by a rectangular hyperbola for each motor, we may assume a hypothetical motor for general calculation having a speed characteristic approximating that of the most modern motors. Such a characteristic would be represented by the following equation to a rectangular hyperbola, coordinating revolutions per minute of the armature and current in terms of fractional parts of the commutation limit;-

$$N = \frac{144}{x} + 330 \text{ --------- } (5)$$

At the commutation limit where $x = 1$, N would equal 474 revolutions per minute. At the average current limit where $x = 0.2$, N would equal 1050 revolutions per minute.

The relation between armature speed in revolutions per minute and train speed on 33 inch wheels with gear ratio G would be

$$N = \frac{S \times 88}{\frac{33}{12} \times \pi} \times G$$

$$N = 10.18 \times S \times G \text{ --------- } (6)$$

Eliminating N between (5) and (6) we have

$$x = \frac{144}{10.18 \times S \times G - 330} \text{ --------- } (7)$$

When $x = 1$, S becomes S_r, the speed of the train on leaving the controller; then

$$S_r = \frac{46.6}{G} \text{ ------------- } (8)$$

Calculating pounds traction per ton — F — from the principles of kinetics and assumed efficiency, at any speed S :–.

$$\text{Output of power in kilowatts} = \frac{F \times V}{737.3} \quad \underline{\text{Principles of Kinetics.}}$$

Input of power in amperes at 500 volts

$$= \frac{F \times S \times 1.467 \times 2}{737.3 \times 0.8}$$

$$I = \frac{F \times S}{201}$$

Pounds traction per ton:

$$F = 201 \frac{I}{S} \text{------------} (9)$$

Calculating required current I_r per ton in accelerating on controller to speed S_r.

From first form of equation (9)

$$I = \frac{F \times S}{201}$$

$$I_r = \frac{330 \times D \times S_r}{201}$$

$$" = 1.64 \times D \times S_r \text{---------} (10)$$

$$" = 1.15 \times D \times S_m \text{--------} (11)$$

Substituting from equation (8)

$$I_r = 1.64 \times D \times \frac{46.6}{G}$$

$$✓ \quad I_r = 76.4 \times \frac{D}{G} \text{---------} (12)$$

Calculating required current per ton at any speed above S_r.

$$I = x \times I_r = \frac{144 \times 76.4 \times D}{G(10.18 \times S \times G - 330)}$$

$$I = \frac{11000\,D}{G(10.18 \times S \times G - 330)} \text{------} (13)$$

The following equation, being a transformation of equation (a) $\underline{\text{Sec. I}}$, gives the acceleration to speed S_r on controller:–

$$\frac{S_r}{T_r} = \frac{300\,D - 11.4}{91.1} \text{-----------} (b)$$

Transposing and substituting from (8)

$$T_r = \frac{91.1}{300D - 11.4} \times \frac{46.6}{G} = \frac{4245}{G(300D-11.4)} \text{-----} (14)$$

Considering now equation (9) giving pounds traction per ton at any current I and corresponding speed S. Ten elevenths of this (as previously explained) minus the train resistance P will give the force available for giving motion of translation to the mass of the train. Thus the acceleration at any instant after leaving the controller, will be represented in miles per hour per second by

$$\frac{\Delta S}{\Delta T} = \frac{\frac{10}{11} \times 201 \times \frac{I}{S} - P}{91.1}$$

but $I = x \times I_r$ and substituting for x and I_r from equations (7) and (12) we have

$$\frac{\Delta S}{\Delta T} = \frac{183\left(\frac{144}{10.18 \times S \times G - 330}\right)\left(76.4 \times \frac{D}{G}\right)}{91.1 \times S} - \frac{P}{91.1}$$

$$\frac{\Delta S}{\Delta T} = \frac{22100 \times D}{S.G(10.18\,S.G - 330)} - \frac{P}{91.1} \text{-----} (15)$$

It is convenient to note from equation (13) that current I equals the first term of this equation (15) multiplied by $\frac{S}{2}$.

The above differential equation if integrated would give an equation to the time-speed curve of a train propelled on a straight level track by means of motors having the assumed speed characteristics and geared and distributed as per G and D.

It is needless to say, that algebraic integration of this equation (involving the element S again, in the expression determining P) would present extreme complication. The differential equation in the form presented offers however, a simple method of integration by calculating the value of $\frac{\Delta S}{\Delta T}$ for the speed on leaving the controller as determined by equation (b) . This will be found to correspond to the result of equation (8) . Now proceeding by "finite differences", an increased speed is found due to acceleration for a small increment of time. For this new speed the value of $\frac{\Delta S}{\Delta T}$ is determined from equation (15), which new acceleration is considered constant for a further small increment of time. This process is repeated in building up the curve.

It is convenient, as a preliminary to building up this curve, to plot the curve represented by the differential equation (15) .

Exhibit H shows curves of time, speed and current calculated from the foregoing formulae for a 100 ton train, with the adhesion corresponding to 50 per cent of the weight on the drivers utilized for traction, and a gear ratio of 2.27 with 33 inch driving wheels.

III. RESISTANCE TO TRAIN MOVEMENT OTHER THAN INERTIA

DEVELOPMENT OF FORMULA FOR TRAIN RESISTANCE.

What by common use is called " train resistance" may be defined as the resultant of the forces opposing the motion of a train when running on a straight level track properly gauged and of standard railway construction, on clean rails, in still air. The elements composing this resultant are varied and no attempt is made herein to analyze the effects of them individually. They consist in part of the friction of journals, wheel flange friction, lack of perfect resilience in the parts under strain of both rolling stock and track, rail joint obstruction and air friction. Were a careful analysis to be made of all these elements their individual relations to the resistance offered to the motion of a train would no doubt be formulated as more or less complicated mathematical functions. As however results are principilly sought by railway operators, which will serve to determine the amount of energy required to maintain speed against the resultant of these various elements, the comparatively simple statement of resistance per unit of mass moved has been the desideratum of the engineer.

Amongst those who have furnished data to the engineering world on the subject as deduced from observations on steam railway practice may be mentioned Clarke, Stroudley, Sinclair, Barnes, Dudley and Wellington. All these engineers made their experiments on comparatively heavy trains. Probably the most scientific determinations were made by Dr. Dudley, and it is to be regretted that so few of his determinations have been published.

In August and September 1898, the writer had occasion to make an extended series of tests on the then newly electrically equipped cars of the South Side Elevated Railway in Chicago. In this connection an opportunity was presented to determine train resistance with light trains under favorable circumstances. About three quarters of a mile of an absolutely level storage track on the elevated structure was given up to the carrying on of the tests. On one of the cars equipped with two motors on one truck, a disk was attached to one of the axles of the trailer truck by means of which a battery circuit was closed once every revolution of the wheel. The circumference of the wheel was calibrated by running carefully over a long measured distance. The revolutions of the wheel were recorded mechanically on a tape; at the same time was marked on the tape time intervals of two seconds by means of mechanism controlled by another battery circuit passing through a clock. This arrangement measured accurately the distance passed over in two second intervals. From this tape record was plotted the various time-speed records shown on exhibits I, J, K, L and M presented herewith.

The deceleration at any point in miles per hour per second multiplied by 91.1 gave the resultant decelerating force in pounds per ton (2000 pounds)

$$F = aM = \frac{S \times 1.467}{t} \times \frac{W \times 2000}{32.2}$$

$$\frac{F}{W} = \frac{S}{t} \times 91.1$$

Such values were first roughly obtained by graphically measuring the value of the tangents at different speeds and plotting them with speeds as abscissae and pounds per ton as ordinates. This was a delicate process but, for the numerous determinations on the single 21 ton car, indicated clearly a straight line with the equation

$$y = 4 + 0.47 \times x$$

Similarly for an 83 ton train consisting of the same motor car with 4 unequipped cars attached, the determined points were coordinated by the equation

$$y = 4 + 0.30 \times x$$

The coefficients 0.47 and 0.3, by simple inspection, might be subject to slight variations and were slightly different in a formula, based on these runs, used by the writer and published in an editorial in the Street Railway Journal about two years ago. By careful reexamination of all available data, the coefficients noted above, not only coordinate, as nearly as inspection justifies, the results on the 21 ton cars and on the 83 ton trains, but they serve to determine coefficients in a general formula on a rational rather than a mere empirical basis, which, more nearly than any other known to the writer, will give results corresponding to scattered data on the subject for all weights of trains, including the results of the writers determinations shown in the accompanying exhibits.

The trains experimented with by the writer were made up as follows:-

1 motor car 20.5 tons with load 0.5 tons-	21 tons
3 motor cars $3 \times 20.5 + 0.5$	62 "
5 motor cars $5 \times 20.5 + 0.5$	103 "
1 motor car 21 tons and 2 trailers $\times$ 15.5	52 "
1 motor car 21 tons and 4 trailers $\times$ 15.5	83 "

It was noted during the time of making the runs that when the trains made up of a motor car and trailers were drifting, no variation was perceptible in the "slack" of the coupling, indicating that the motor car and trailers were decelerating independently at the same rate. This is borne out by the results shown on the exhibits, the trains being of varied composition and evidently subject to variation of train resistance as a function of weight independently of composition.

It might have been thought, with trains composed of motor cars alone, that, owing to the inertia of the revolving armatures, deceleration might have been less rapid than with unequipped cars. On the other hand it might have been thought that , owing to gear friction, (the armature shaft being connected to the axle through single reduction gearing) the deceleration might have been more rapid than with unequipped cars. The writer can simply make the statement that, within the limits of observation, these elements balanced throughout the experiements made by him, and it is not unreasonable to consider that within working limits the composition of an electric train may be neglected in considering train resistance.

Considering now that, with a hypothetical zero weight of train, the resistance per ton would be infinity, we have the following relations between weight and train resistance:-

0 Tons, Resistance = $4 + S \times \infty$ lbs per ton

21 " " = $4 + S \times 0.47$ " " "

83 " " = $4 + S \times 0.30$ " " "

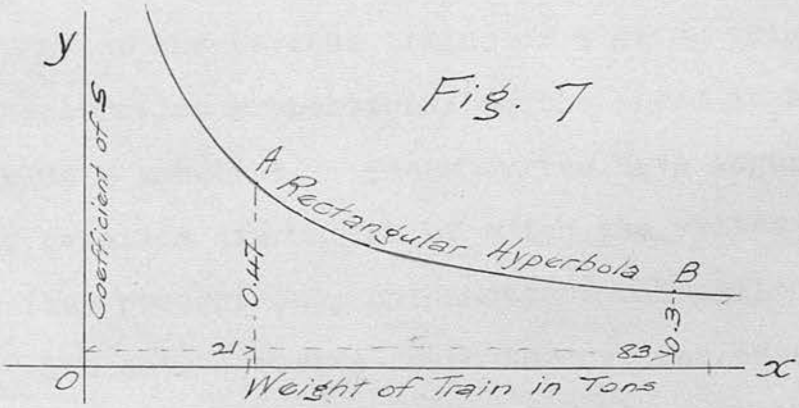


Fig. 7

Plotting now (fig 7) with weights (W) as abscissae and coefficients of S as ordinates, a rectangular hyperbola may be passed through the two points A and B, having as an asymptote the axis of y

$$x \times (y - y_1) = C \text{ - - - - - - } (c)$$

$$21 \times (0.47 - y_1) = C$$

$$83 \times (0.30 - y_1 = C$$

$$98.7 - 21y_1 = 24.9 - 83y_1$$

$$\underline{y_1 = 0.24 \text{ Approx.}}$$

$$\underline{C = 21 \times (0.47 - 0.24) = 4.8 \text{ Approx.}}$$

From (c) - - - - $y - y_1 = \frac{C}{x}$

$$y - 0.24 = \frac{4.8}{x}$$

but, Coefficient of $S = y$

$\therefore$ Coefficient of $S = 0.24 + \frac{4.8}{W}$

Thus we have the general formula relating Train Resistance in pounds per ton (R), Speed in miles per hour(S) and weight of train in tons (2000 pounds) W

$$\underline{R = 4 + S\left(0.24 + \frac{4.8}{W}\right)} \text{ - - - - } (16)$$

According to this formula the time-speed drifting curve of any of the various trains of a given weight, will have a deceleration proportional to the speed it has at the instant plus a constant. These curves have logarithmic equations, detailed discussion of which the writer does not offer as they present only mathematical curiosities.

The actual curves drawn through the observed results on <u>Exhibits I, J, K, L and M</u> were integrated by the convenient process of finite differences i. e., assuming any speed; the deceleration due to the resistance given by the formula for the given weight, is calculated. This deceleration is considered as constant for a few seconds, to a lower speed. This process is repeated in building up the curve.

The formula is thus seen to conform, within ordinary limit of error, to all the observations.

As additional confirmation of the correctness of the formula, the writer , within a few weeks of the present writing, had the opportunity, in the course of making service tests of electric equipment on the Brooklyn Elevated Railway, to make a drifting test on a train weighing almost exactly 80 tons, consisting of a heavy motor car and three unequipped cars. The recording apparatus was exceptionally delicate; automatically recording distance in revolutions of the wheel and time in half seconds. In descending a long straight gradient of 0.7 per cent at a speed of 32 miles per hour no perceptible variation in speed was recorded, indicating a gravity pull of 14 pounds per ton exactly compensating for train resistance. The formula for this speed and weight of train gives 13.6 pounds per ton.

Two of Dr. Dudley's tests of resistance on steam trains, are as follows:-

1- Weight 313 tons. Speed 51.4 m.p.h. Resistance 16.9 lbs.

2- Weight 242.5 tons Speed 63.5 m.p.h. Resistance 19.8 lbs.

The formula gives for No.1, 17.1 lbs per ton and for No. 2, 20.5 lbs per ton.

RESISTANCE DUE TO GRADIENTS AND CURVATURE.

Were a train to continue running on a reentrant track, like those of the Metropolitan "Circles" in London, having energy supplied for overcoming all resistance to motion except that presented by gravity, then, in a round trip from any point on the line to the same point again, gravity would alternately give kinetic energy to the mass on descending gradients and store an equivalent amount in a potential form on ascending gradients. This however, is is not a working railway condition. Stops must be made at points to a great extent independent of position on general gradient, and energy dissipated in so doing which might otherwise be utilized in useful work.

There are situations, such as deep level tunnels like that of The Central London Railway, where the stations may be so situated that advantage may be taken of descending gradients in accelerating and of ascending gradients in decelerating.

In the Central London Double Tube Tunnel, starting gradients are 1 in 30 , and stopping gradients (for precautionary reasons) 1 in 60. This has proved an admirable arrangement and distinctly a means of economizing energy required for train movement.

Surface or Elevated lines must conform to a great extent, to the nature of the variations in elevation of the territory they serve, and for this reason gradients may not compensate in the balancing of energy of position and energy of motion, as they otherwise might.

In accelerating and running at more or less uniform speed, energy must be supplied for climbing all gradients, while on descending gradients gravity can be utilized only to such an extent as will not give undue speed.

Over the distance in which the brakes are applied an ascending gradient is taken advantage of in partially transforming the kinetic energy of the train into energy of position. On the other hand, in stopping on a descending gradient, energy of position as well as the kinetic energy of the train must be dissipated by the brakes, except as such energy serves to carry the train over the distance in which the brakes are applied. Thus, in accurately determining effects of gradients on train movement, each run between stations would require separate consideration.

The modification of energy requirements resulting from gradients, on such train movement as is herein considered (unless in special cases like that of Central London) will not justify too great refinement of calculation. The writer has found the following rule for calculation of the work to be done on account of gradients to give results as close as can be verified by service tests;-

In a round trip;

1- Calculate the work required to raise the mass through the height covered by all ascending gradients.

2- Deduct the work represented by lowering the mass through the height covered by all descending gradients up to 0.75 per cent.

3- The work represented by lowering the mass through the height covered by the excess in descending gradients over 0.75 per cent is not to be deducted, it being considered as dissipated in the brakes as a precautionary measure in preventing excessive speed.

It will be noted that the above rule permits neglect of consideration of the effect of gradients in calculating energy requirements on lines where the gradients do not exceed 0.75 per cent or about 1 in 133. Where however gradients are as high as two per cent and over, as they are on the elevated lines in New York, the energy requirement for gradients may reach ten per cent of the total energy reqired for movement.

The resistance offered to the motion of a train by curvature in the track, the writer has found to be in his experiments a more or less uncertain quantity, owing principally to track conditions. The super-elevation of the outer rail is correct only for one speed. Again, many sharp curves are improperly gauged for the rigid wheel bases they have to accommodate, so unduly binding the wheels.

During the tests already referred to as made by the writer on the South Side Elevated Railway in Chicago, only one good curve was available for making curvature resistance tests. This was of 140 feet radius and about 220 feet long. On approaching this curve with a single car at a speed of from 20 to 25 miles per hour, first there was an apparent sudden dropping off in speed, partially due presumably to giving the mass angular velocity around a vertical axis, then, a comparatively uniform deceleration of about 0.6 miles per hour per second. This would correspond to a total resistance per ton (2000 pounds) of about 55 pounds. Deducting so called "train resistance" say 14 pounds, there remained 41 pounds per ton due to resistance presented by the curve.

Considering resistance per ton P, as proportional to curvature we have (R being radius in feet)

$$P = C \times \frac{1}{R}$$

$$C = P \times R = 41 \times 140$$

$$C = 5740$$

This constant develops a very convenient expression for the work to be performed against curvature. Calling A the total curvature in degrees passed over in making a round trip of the line, then the energy required to overcome curvature in a round trip in feet pounds per ton (F) would be

$$F = \Sigma \frac{5740}{R} \times 2\pi R \times \frac{A}{360}$$

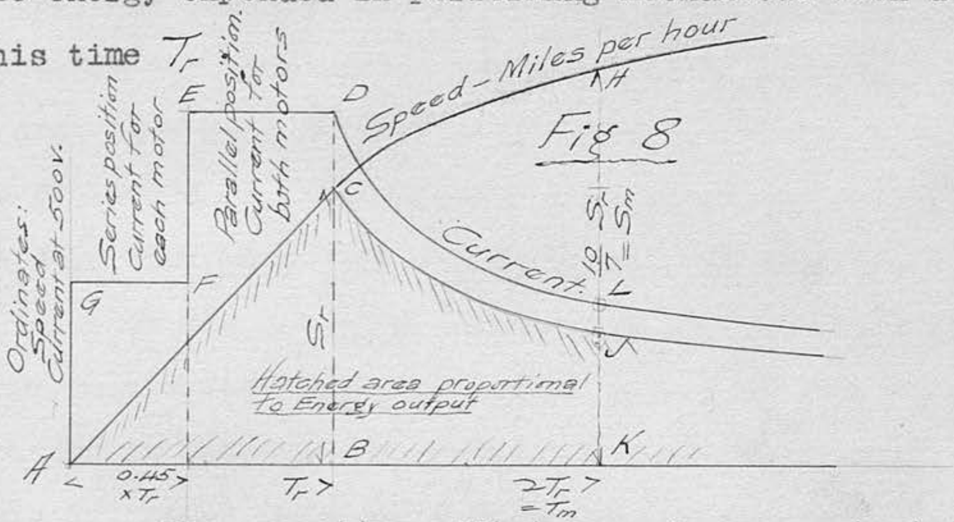
$$F = 100 A \text{ in foot pounds} \quad \text{---} (17)$$

The result of the preceeding test is the only experimental datum on the subject the writer has been able to obtain- Reference may however be made to accepted American practice in railway location. The standard compensation for curvature in modifying gradients, as the result of experience in adapting locomotives to the loads they are required to haul, is that for each degree of curvature in 100 feet a reduction of 0.04 per cent be made in gradients on which the curves exist. This corresponds to an assumed curvature resistance of 0.8 pounds per ton (2000 pounds) per degree in 100 feet. ------ The experimental result noted above corresponds to about one pound per ton per degree in 100 feet, or a somewhat greater amount than that allowed in standard railway curvature compensation, but as sharper curves are to be dealt with in light suburban lines than in trunk lines, as the work to be done in overcoming curvature is a small fraction of the whole, as the greater coefficient corresponds to an experimental result and errs if any on the safe side, and as it develops a round figure formula, the writer adopts it in his calculations of work to be done against curvature.

IV. POWER RATING OF MOTORS AND ENERGY REQUIRED FOR TRAIN MOVEMENT.

PRELIMINARY.

There being during the time of operation on the controller a constant traction, the power output at the driving wheels will vary directly as the speed, thus the area ABC shown on fig. 8 will measure, with a proper coefficient the energy expended in performing mechanical work during this time T_r



The operating efficiency of power output to power input at 500 volts being assumed at 80 per cent, power input may be represented at time T_r by an ordinate $BD = \frac{100}{80} CB$. This ordinate will also represent to a convenient scale current input, so that the area ABDEFG represents energy input to the same scale as ABC represents energy output. Thus the ratio of energy output during operation on the controller, represented by the area ABC, to energy input during the same period, represented by the area ABDEFG, is, calling Z the power proportional to the ordinate BC,

$$= \frac{Z \times \frac{T_r}{2}}{(1.25Z \times 0.55T_r) + (\frac{1.25}{2}Z \times 0.45T_r)}$$

$$\text{Ratio} = 0.515$$

= Energy efficiency during acceleration on the controller.

During the continued propulsion of the train, the ratio of energy output to energy input will be 0.8, in accordance with our efficiency assumption; thus, in fig 8 the ordinates from BC toward KJ being 0.8 of the ordinates from BD toward KL, will represent power output. The area AKJC will represent energy output for any time AK.

We are now in a position to consider the varying energy requirement for the general method of train movement discussed in Section I. wherein, in order to compare the possibilities of movement under varying conditions of distance, time, speed and acceleration we considered;-

1st- The period of acceleration to be twice that of operation on the controller and the speed $-S_m-$ then attained ten sevenths of that $-S_r-$ on leaving the controller. (S_m is conveniently called average full speed)

2nd- The train to run at that uniform speed until the application of the brakes.

3rd- Braking from that speed at the average rate of 1.5 miles per hour per second.

Referring to fig. 8, the following relations between ordinates and areas will be found to exist.

Area AKLDEFG $= 0.69 \times AK \times BD$

If for AK we substitute T_m and for BD we substitute I_r the area will represent with the coefficient 500/1000 (line p.d being 500 volts) the energy in kilowatt seconds per ton of train mass accelerated to average full speed S_m under the conditions shown.

Energy in kilowatt-seconds for total acceleration to $S_m = \frac{500}{1000} \times 0.69 \times T_m \times I_r$

Substituting from formula (3) $T_m = \frac{42.6\, S_m}{100D - 3.8}$

and from formula (11) $I_r = 1.15\, D \times S_m$

We have energy (E) in kilowatt-seconds for total acceleration to S_m, in terms of average full speed (as per kinematic diagram) and the fraction of weight utilized for traction:- $0.5 \times 0.69 \times \frac{42.6 \times S_m}{100D - 3.8} \times 1.15 \times D \times S_m$

$$E = 16.9 \times S_m^2 \times \frac{D}{100D - 3.8} \quad ---- (18)$$

From the train movement diagram we note S_m and D for any given conditions. From the same diagram we note the distance required for acceleration and deceleration; this deducted from the total length of run will give the distance to be run at average full speed. The train resistance formula will give the resistance to be overcome, in pounds per ton of train mass at this speed. The foot-pounds of work to be performed against this resistance may be found by multiplying the distance run at average full speed by the resistance in pounds. Foot-pounds output are reduced to kilowatt-seconds input by dividing by $738 \times 0.8 = \underline{590}$ (considering efficiency at 80 per cent). It will be noted that the energy expended against train resistance during acceleration is absorbed in the formula for energy of acceleration. The remaining element of work to be done (viz: that against gradients and curvature) may be calculated, as hereinbefore explained, in foot-pounds and reduced to kilowatt-seconds input as in the case of energy expended against train resistance.

We have thus developed the following simple method of calculation (giving results justified by service tests) of electric energy input required for any given conditions of train operation, with short distances between stations and comparatively high speeds-

First, determine the average station distance; then, for any suitable schedule time of running, determine from lengths of station stops and other so called "dead time" the time allowed for making an average unrestricted run between stations. Refer now to the general train movement diagram for the kinematics of movement under varying accelerations for the given distance and time. From this diagram the factors S_m and D are obtained, and also the distance of running at average full speed.

The energy required during acceleration, that required to overcome train resistance at average full speed, and the average energy per station run required to overcome gradients and curvature, will, together, give the average input of energy required per run. This average energy input per run divided by the time of making an average run plus the average time of stop and other dead time, will give the average power input required, which is an important factor in the selection of motors, as elsewhere explained. This average power is also a most important factor in determining Transmission Line and Power Station capacity. The subjects of Transmission Line and Power Station are however not treated of in this thesis.

THE ECONOMIC SELECTION OF A MOTOR.

In selecting a motor for any specific service the relations between existing commercial horse power rating and the requirements of commutation and thermal capacity, as before explained, may be reduced to formulae as follows:-

I_r = The working commutation limit per ton, being 25% in excess of commercial full load current.

K = The average input in kilowatts referred to in text being 25% in excess of commercial rated load.

$H.P.$ = Horse power of motors per ton of train mass-commercial rating.

H-P as determined by commutation limit

$$= \frac{0.8 \times I_r \times 500 \times 0.8}{1000 \times 0.746}$$

$$= 0.43 \times I_r = \underline{0.5 \times D \times S_m} \text{ - - - - } (19)$$

H-P as determined by temperature limitation of motor windings

$$= \frac{4 \times K \times 0.8}{0.746} = \underline{4.3 \times K} \text{ - - - - } (20)$$

By now plotting for any average train speed under consideration, the results of equation (19) for corresponding values of D we have a curve showing increasing horse power capacity of motors required with increasing fractions of the weight on the drivers utilized for traction.

Again for the same average train speed, plotting the results of equation (20) for corresponding values of D we have a curve showing decrease in horse power capacity of motors required with increasing fractions of weight on the drivers utilized for traction.

Where these two curves intersect we obtain the minimum motor capacity required for the performance of the service indicated and also the necessary distribution of weight on the driving wheels for the same.

The writer has found in making detailed estimates on proposed installations, that the method of movement determined by this point of minimum motor equipment, is the most economical that can be selected as the basis of deter-

mination of the complete Electric Railway Plant, both as to first cost and as to subsequent cost of operation.
Proof of this statement involves consideration of traffic requirements, corresponding train service, variations in train weight, first cost of plant and equipment, and subsequent cost of operation including maintenance, depreciation, fixed charges, etc. No attempt is made herein to demonstrate this general proposition.

The following example will render more clear the methods of calculation demonstrated;-

Average Train weight ------------- 100 Tons (2000)

Average distance between stations 1800 Feet.

Average unrestricted running speed between stations;-

A ------------ 17 Miles per hour.

B ------------ 19 " " "

C ------------ 21 " " "

Average running time from start to stop between stations;-

A ------------ 72.2 Seconds

B ------------ 64.6 "

C ------------ 58.5 "

Average "dead time" per station run --- 20 Seconds.

Average schedule time from start to start between stations;-

A ------------ 92.2 Seconds.

B ------------ 84.6 "

C ------------ 78.5 "

Average schedule speeds;-

A ------------ 13.31 Miles per hour.

B ------------ 14.52 " " "

C ------------ 15.63 " " "

In the tabulations which follow, the figures in the various lines represent the following;-

(a) Percentage of weight of train utilized for traction.

(b) Average maximum speed in miles per hour. From diagram, Sec.I

(c) Combined distance of accelerating and decelerating. "" "

(d) Distance - uniform running in feet.

(e) Train resistance in pounds per ton ----- $4 + S_m\left(0.24 + \frac{4.8}{W}\right)$

(f) Energy per ton for Acceleration ---- Kilowatt-seconds $16.9 \times S_m^2 \times \frac{D}{100D - 3.8}$

(g) " " " Uniform running " " $\frac{(d) \times (e)}{590}$

(h) " " " Gradients & Curvature " " (Determination for Manhattan El. Ry.)

(j) " " " per average run between stations.

(l) Average power per ton ----- Kilowatts $= \frac{(j)}{\text{Schedule time per average run}}$

(m) Horsepower of Motors per ton as determined by commutation limit.

$$= 0.5 \times D \times S_m$$

(n) Horsepower of Motors per ton as determined by temperature limitation ----- $4.3 \times (l)$

NOTE. Calculations are only correct within the limit of 10 inch slide rule error.

A --- Average unrestricted running speed between stations;-

17 Miles per hour.

(a)	30	35	40	50	60	80	I00
(b)	23.9	22.3	21.5	20.5	20.0	19.4	19.1
(c)	1210	930	790	610	530	420	370
(d)	590	870	1010	1190	1270	1380	1430
(e)	10.9	10.5	10.2	10.0	9.8	9.6	9.5
(f)	111	94.5	86	76.5	72.2	66.8	64
(g)	11	15.5	17.5	20.1	21.1	22.5	23
(h)	9	9	9	9	9	9	9
(j)	131	119	112.5	105.6	102.3	98.3	96
(l)	1.42	1.29	1.22	1.14	1.11	1.07	1.04
(m)	3.58	3.91	4.30	5.11	6.00	7.78	9.55
(n)	6.12	5.56	5.25	4.91	4.78	4.60	4.48

B --- Average unrestricted running speed between stations;-

19 Miles per hour.

(a)	40	50	60	80	100
(b)	26.7	24.7	23.7	22.7	22.2
(c)	1210	890	740	580	510
(d)	590	910	1060	1220	1290
(e)	11.8	11.2	10.9	10.6	10.4
(f)	133.2	111.6	101.5	91.6	86.7
(g)	11.8	17.2	19.5	21.9	22.8
(h)	9	9	9	9	9
(j)	154.0	137.8	130.0	122.5	118.5
(l)	1.82	1.63	1.54	1.45	1.40
(m)	5.34	6.18	7.11	9.10	11.10
(n)	7.82	7.00	6.62	6.23	6.02

C --- Average unrestricted running speed between stations;-

21 Miles per hour.

(a)	50	60	70	80	100
(b)	30.6	28.4	27.3	26.6	25.7
(c)	1370	1060	910	800	680
(d)	430	740	890	1000	1120
(e)	12.9	12.2	11.9	11.7	11.5
(f)	171.0	145.5	133.2	125.7	116.0
(g)	9.3	15.4	18.0	19.9	21.8
(h)	9.0	9.0	9.0	9.0	9.0
(j)	189.3	169.9	160.2	154.6	146.8
(l)	2.40	2.16	2.04	1.97	1.87
(m)	7.65	8.52	9.55	10.63	12.85
(n)	10.31	9.30	8.77	8.47	8.04

The last two lines of each of the foregoing tabulations are plotted on <u>Exhibit N</u>, in the manner previously explained. Abscissae are percentages of weight of train utilized for traction in accelerating, and ordinates are commercial horsepower of motors per ton of train. The horsepower as determined by limit of commutation increases with increasing acceleration, while the horsepower as determined by temperature limitation of the motor windings decreases with increasing acceleration, as hereinbefore demonstrated.

For each average train speed there are thus two curves plotted which intersect in a well defined minimum of motor capacity. At this point we obtain for the average train speed indicated, the economic selection of motor capacity, and also the required distribution of motors to give the necessary weight on the driving wheels.

V. FLUCTUATIONS OF STATION LOAD DUE TO TRAIN MOVEMENT.

PRELIMINARY.

With a maximum current requirement per train greatly in excess of the average requirement, the probable fluctuations of station load have presented a problem more or less difficult of solution, depending on the method of attacking it.

Given any railway situation with all its variations of station distance, arbitrary restrictions to free running, dead time etc., we would have, under normal conditions of operation , with one train only in continual circuit, a regular cycle of current demand in a "round trip". This may be considered an elementary time-current rhythmic function . Consider now trains, as in service, running at definite time intervals apart, each developing the same or similar rhythmic current demands, we would have a combination time-current rhythmic demand on the Power Station feeding the section, which would be some regularly recurring function.

The writer has made attempts to attack the problem of determining the limits of this rhythmic cycle of current demand on the Power Station by consideration of the theory of probabilities. The analytical methods known to the writer produced however no satisfactory determination of the probable extent of these fluctuations.

The following graphical method was then applied:- With time as abscissae was plotted a complete cycle of current demand for unit train mass making a complete circuit of the line under service conditions, over a given railway with its varying station distances etc. Considering now the time intervals at which trains were supposed to be started: such intervals were stepped off throughout the time of making a complete circuit of the line, and at each point so stepped off a note of the current demand there shown was made. The sum of the current readings in stepping off a complete circuit, represented the total current demand, for the instant, on the Station; the average number of trains in circuit (probably fractional) being determined by dividing the total time of a train in circuit (including all dead time at termini and otherwise) by the time interval between trains.

Repeating the stepping process, say one second later, a new sum was obtained representing the total current demand at that instant. This process was repeated throughout a starting interval, which determined the time of a complete cycle of variation of current demand. The process was a tedious one but, by repeating it under different conditions of maximum and average current, on varying time intervals of starting and varying station distances, a solution of the problem of the probable extent of fluctuations on the Power Station gradually became apparent. The writer kept an assistant at work on this graphical determination of fluctuations for several months, when opinion as to probable excessive fluctuations on the proposed Illinois Central Railway electric installation, seemed the most powerful argument that could be brought to bear against the economy of rapid acceleration.

APPARENT LAW OF FLUCTUATIONS.

From the foregoing determinations, together with further graphical demonstration on well defined conditions of operations, and also by continued observation of actual Station fluctuations, the writer considers the following law of fluctuations as sufficiently well defined for determining the probable variations of current which may be expected on a Power Station designed for given conditions of train service:-

> The maximum fluctuation of current which may be expected, under normal conditions, on an Electric Railway Power Station, will vary above or below the average current requirement, not to exceed twice the maximum demand of the average train, independently of the number of trains in circuit.

In illustration of this law, Exhibit O represents the calculated fluctuations (by the method explained) caused by varied numbers of trains in circuit, on a hypothetical substation feeding a section of the Manhattan Elevated Railway in New York (still unequipped electrically). A plan of the line and a diagram of the current requirement per train for an average run between stations, under the conditions noted, are shown on Exhibit P. The maximum power demand per train as noted, is 1300 kilowatts, while, no matter what the number of trains in circuit, the fluctuations of load on the Station above or below the average load, do not exceed twice the maximum demand per train or 2600 kilowatts except in one case viz: with 88 trains in circuit at starting intervals of 30 seconds a maximum fluctuation above average of 3200 kilowatts as noted. The average load on the station during this condition of operation is 24552 kilowatts.

In confirmation of the enunciated law, Exhibits Q,R and S show the actually recorded fluctuations on the generating station and on two outlying battery substations feeding the South Side Elevated Railway in Chicago during a "rush" hour. This railway is operated on a distributed motor system with about 50 per cent of the weight of the train on the drivers; each car being equipped with two motors on one of its trucks. A plan of this line is shown on Exhibit T., together with the current requirement per train for an average run between stations under the conditions noted. The maximum current demand per train was about 700 amperes. It will be noted from the station fluctuations, that no matter what the average load on the station, the fluctuation above or below average load, did not exceed twice the maximum current demand per train, or 1400 amperes.

FINIS.

John Lundie

Nov 1/1901

EXHIBIT A.

Rapid acceleration run at Schenectady, Aug. 1897.

Copy of First Rapid Acceleration Record
Made by the General Electric Company
At Schenectady, August 4th, 1897,
For John Lundie, Consulting Engineer,

Two G.E. 57 Motors.

Weight of Car, 8 tons.

Weight of Passengers 1 ton

Total Weight 9 tons.

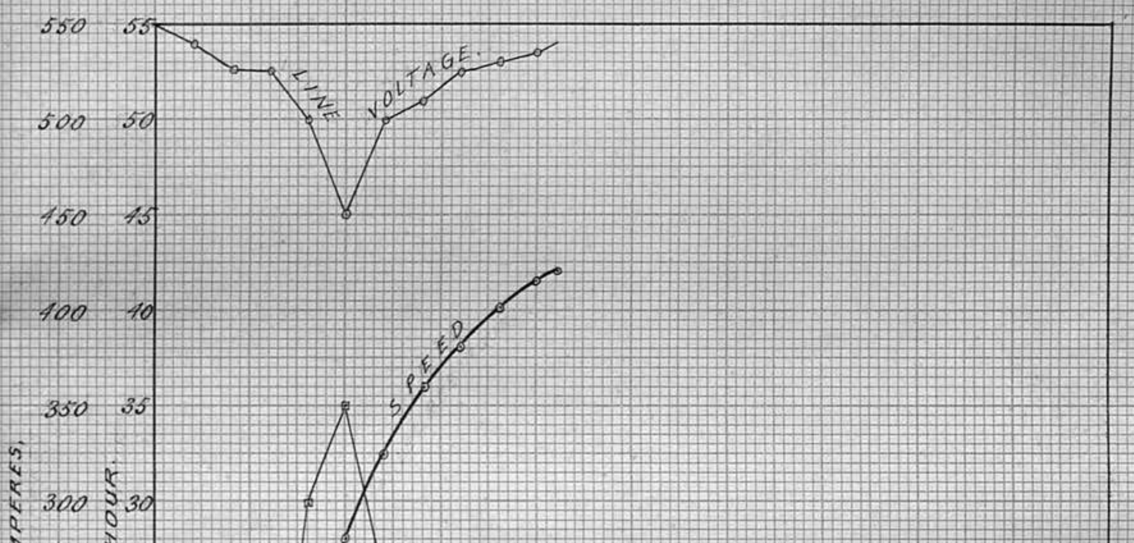


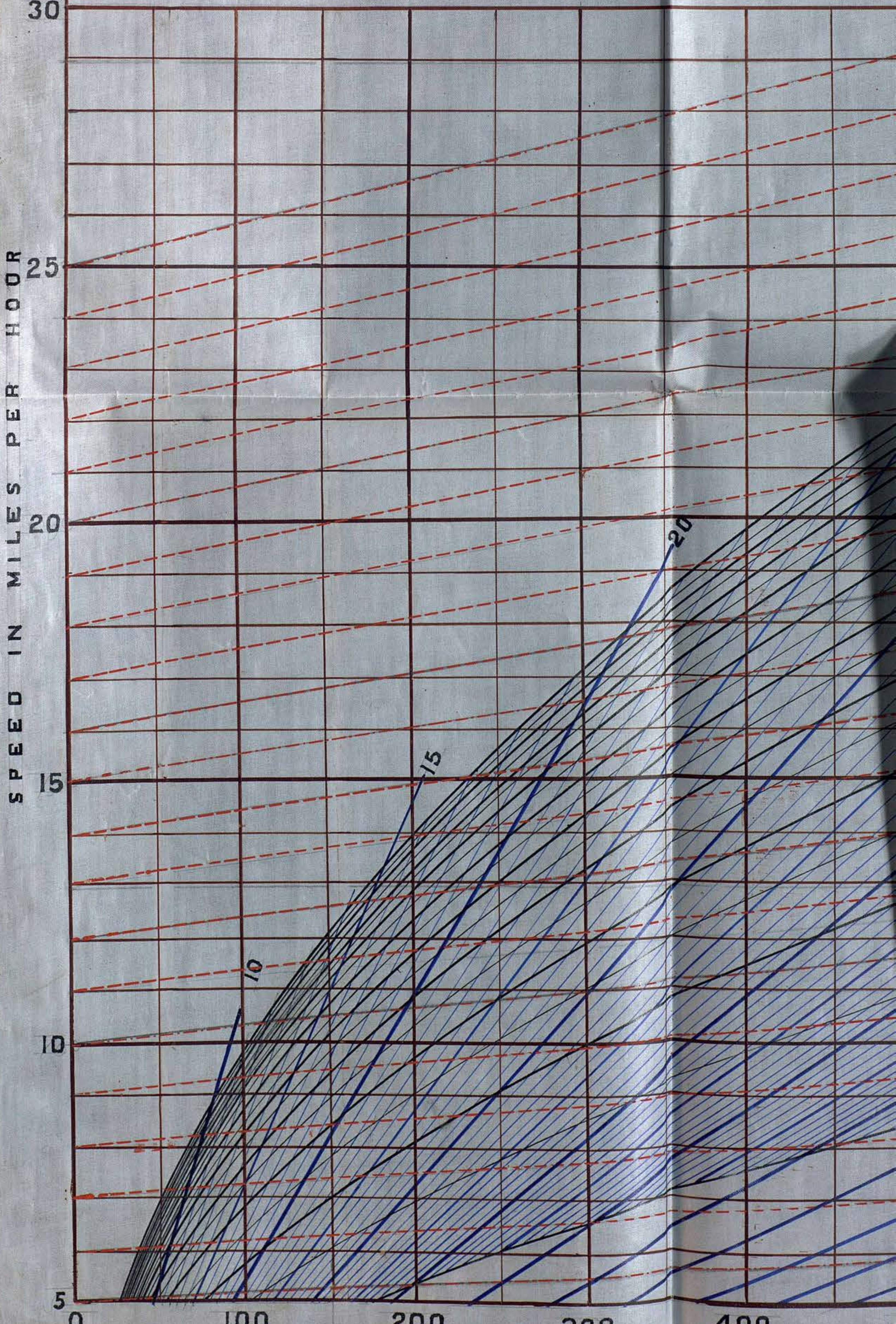
EXHIBIT B.

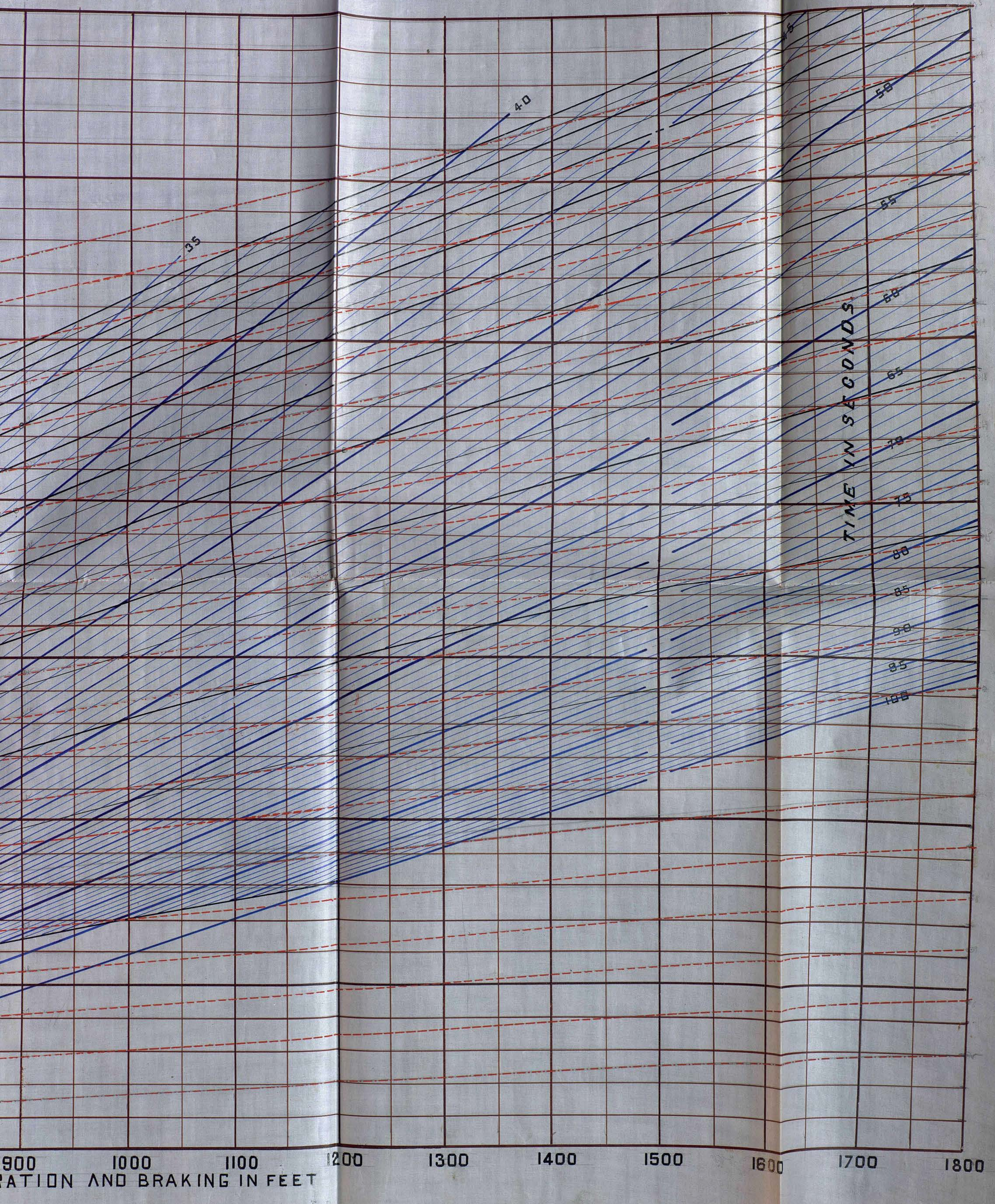
Train Movement Diagram.

TRAIN MOVEMENT DIAGRA

Designed by John Lundie.

Ordinates:
Speed in miles per hour
Abscissae:
Combined distance in feet of accelerating and decelerating
Total length of run for given setting of average speed lines.
Blue Time Lines:
Relation between time and distance of accelerating and decelerating with varying m
Red Average Speed Lines:
These lines vary in inclination for each total length of run. The ordinates of any point on th
give the maximum speed corresponding to the acceleration indicated by the curved line
point, with time and distance required to accelerate and decelerate; the remainder, if any,
total run being made at this maximum speed.
Black Curved Acceleration Lines:
Loci of possible acceleration as dependent on percentage of weight available fo
adhesion being taken at 15% of this, and a uniform acceleration on this basi
maximum speed - remainder on series motor curve. Deceleration taken throughout a
per hour per second on curve similar to that of acceleration.




35
40
45
50
55
60
65
70
75
80
85
90
95
100
TIME IN SECONDS
900
1000
1100
1200
1300
1400
1500
1600
1700
1800
RATION AND BRAKING IN FEET

TRAIN MOVEMENT DIAGRAM,

Designed by John Lundie.

Ordinates:

Speed in miles per hour

Abscissae:

Combined distance in feet of accelerating and decelerating

Total length of run for given setting of average speed lines.

Blue Time Lines:

Relation between time and distance of accelerating and decelerating with varying maximum speed.

Red Average Speed Lines:

These lines vary in inclination for each total length of run. The ordinates of any point on these lines give the maximum speed corresponding to the acceleration indicated by the curved lines at that point, with time and distance required to accelerate and decelerate; the remainder, if any, of the total run being made at this maximum speed.

Black Curved Acceleration Lines:

Loci of possible acceleration as dependent on percentage of weight available for traction, adhesion being taken at 15% of this, and a uniform acceleration on this basis to 0·7 of maximum speed – remainder on series motor curve. Deceleration taken throughout at 1·5 miles per hour per second on curve similar to that of acceleration.

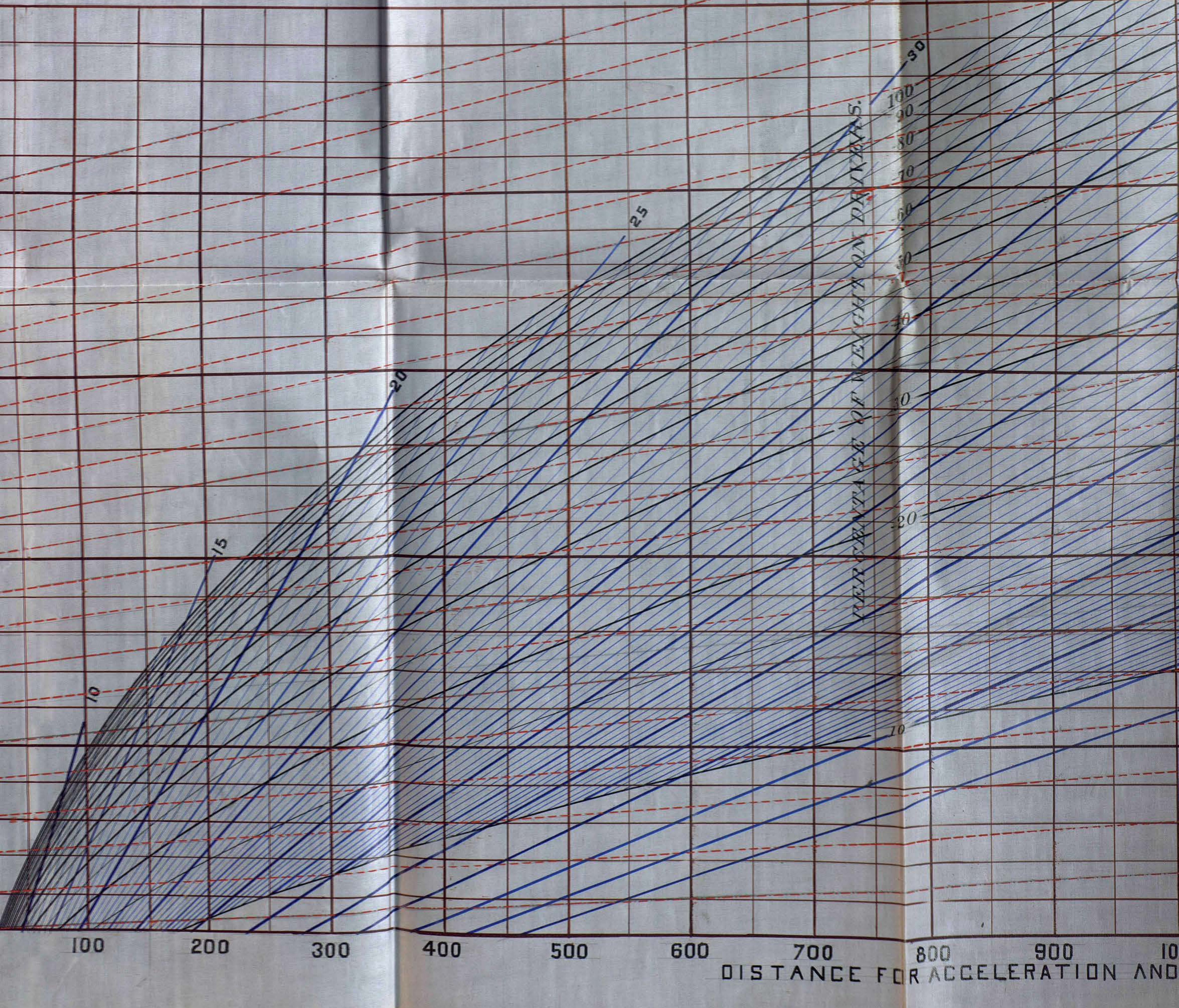
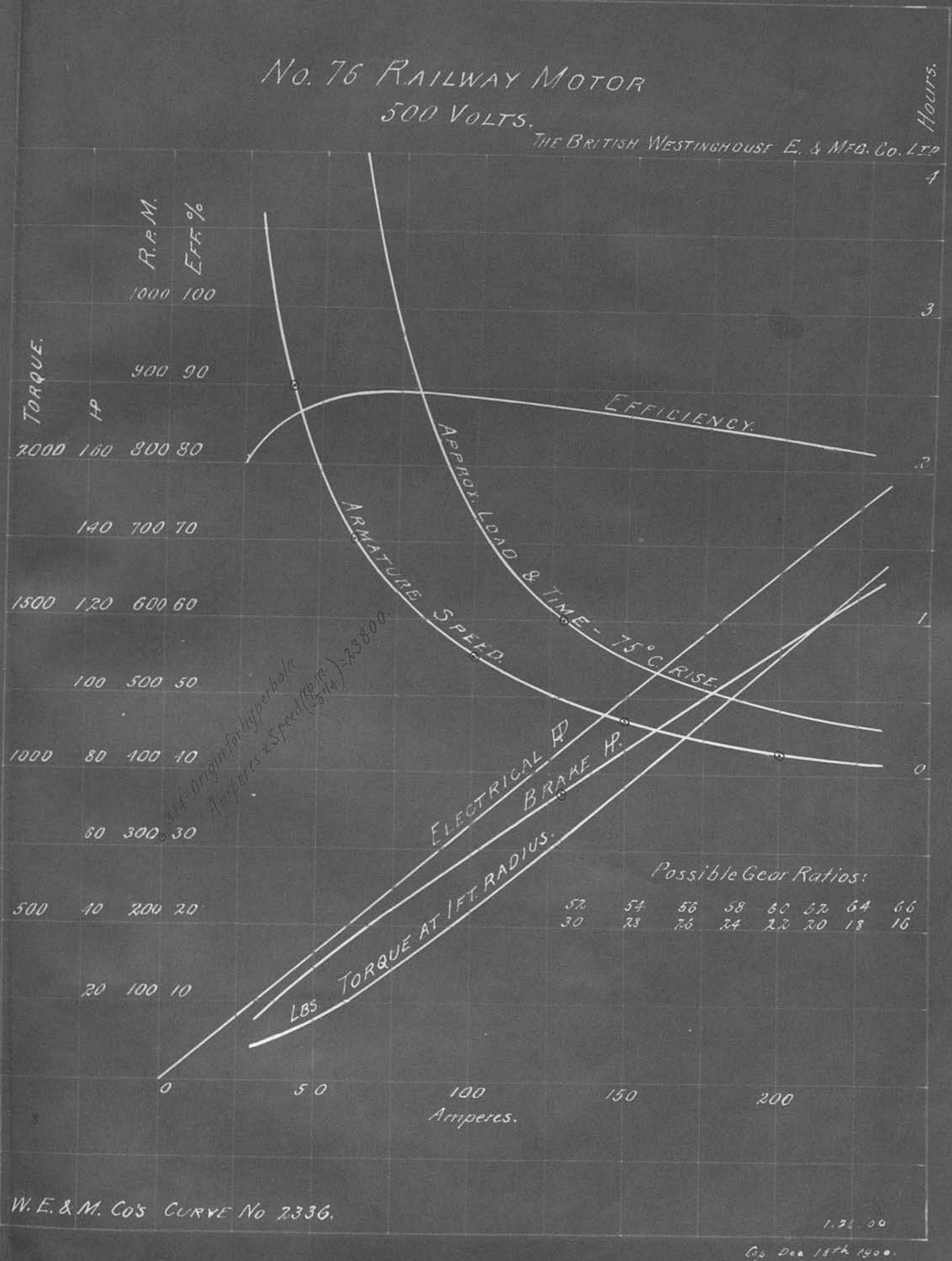


EXHIBIT C.

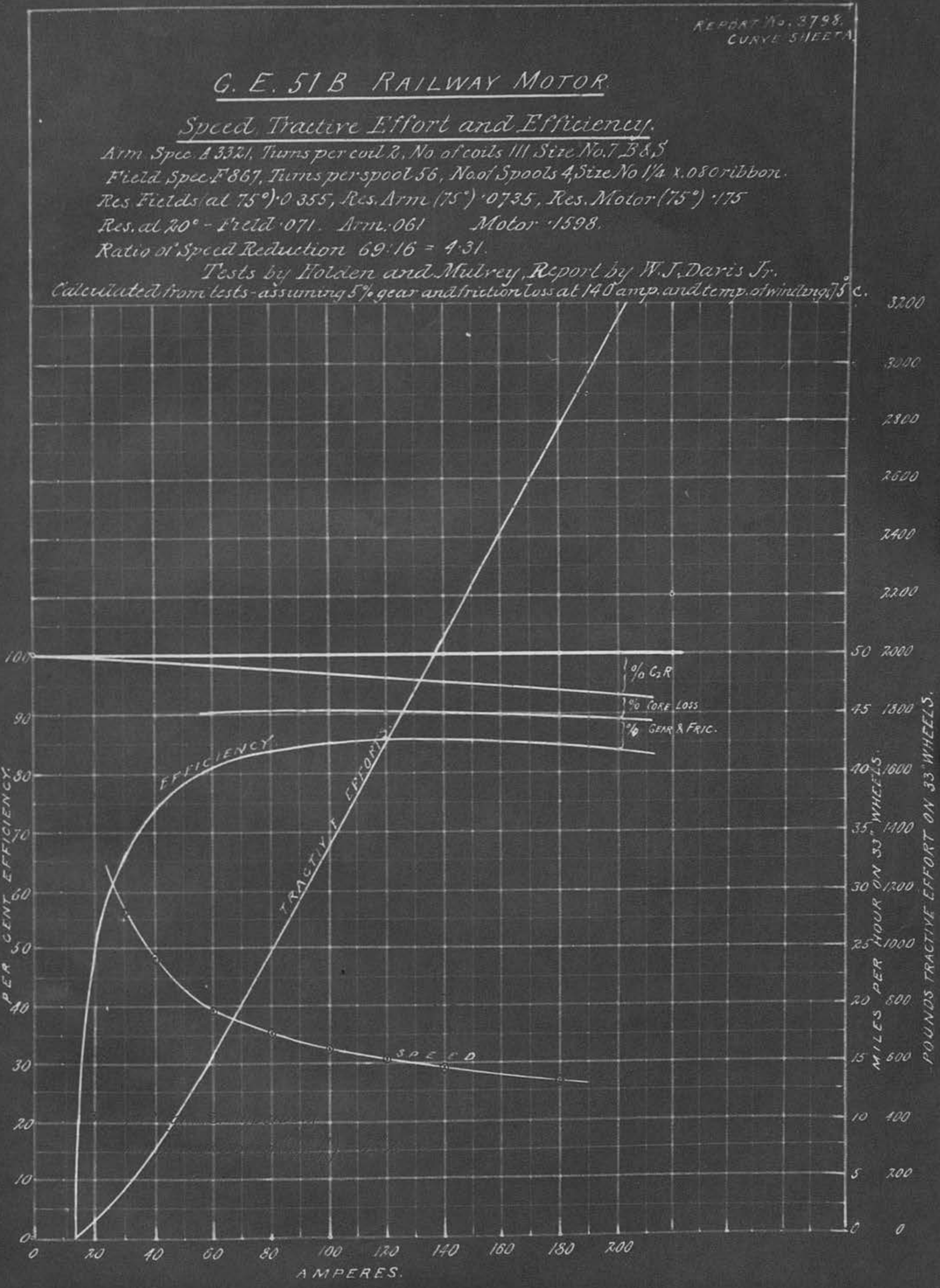
Westinghouse No 76 Motor characteristic sheet.



No. 76 Railway Motor
500 Volts.
The British Westinghouse E. & Mfg. Co. Ltd
Hours.
Torque.
HP
R.P.M.
Eff. %
1000 100
900 90
2000 160 800 80
140 700 70
1500 120 600 60
100 500 50
1000 80 400 40
60 300 30
500 40 200 20
20 100 10
0
50
100
150
200
Amperes.
4
3
2
1
0
Efficiency.
Approx. Load & Time - 75° C. Rise.
Armature Speed.
Electrical HP
Brake HP.
Lbs Torque at 1 ft. Radius.
314 = origin for hyperbola
Amperes × Speed (r.p.m. / 3.14) = 23800.
Possible Gear Ratios:
52 54 56 58 60 62 64 66
30 28 26 24 22 20 18 16
W. E. & M. Co's Curve No 2336.
1.26.00
Cop. Dec 18th 1900.

EXHIBIT D.

General Electric Co. No. 51 Motor characteristic sheet.



REPORT No. 3798.
CURVE SHEET A
G. E. 51 B RAILWAY MOTOR
Speed Tractive Effort and Efficiency.
Arm Spec. A 3321, Turns per coil 2, No of coils 111 Size No.7 B&S
Field Spec F867, Turns per spool 56, No. of Spools 4, Size No 1/4 x .080 ribbon.
Res Fields (at 75°) ·0355, Res. Arm (75°) ·0735, Res. Motor (75°) ·175
Res. at 20° - Field ·071. Arm. ·061 Motor ·1598.
Ratio of Speed Reduction 69:16 = 4·31.
Tests by Holden and Mulrey, Report by W. J. Davis Jr.
Calculated from tests - assuming 5% gear and friction loss at 140 amp. and temp. of winding 75° C.
% C_2R
% CORE LOSS
% GEAR & FRIC.
EFFICIENCY
TRACTIVE EFFORT
SPEED
PER CENT EFFICIENCY
MILES PER HOUR ON 33" WHEELS
POUNDS TRACTIVE EFFORT ON 33" WHEELS
AMPERES.
0 20 40 60 80 100 120 140 160 180 200
0 10 20 30 40 50 60 70 80 90 100
0 5 10 15 20 25 30 35 40 45 50
0 200 400 600 800 1000 1200 1400 1600 1800 2000 2200 2400 2600 2800 3000 3200

EXHIBIT E.

Tabulation of Motor speed characteristics.

RATING AND SPEED CHARACTERISTICS OF THE PRINCIPAL MODERN COMMERCIAL RAILWAY MOTORS

Suitable for heavy work, manufactured in the United States of America, October 1901.

Compiled, reduced and co-ordinated by John Lundie.

	Westinghouse #49.	G.E 1000	Westinghouse #68	Westinghouse #38	G.E #57	Westinghouse #56	Westinghouse #76	G.E #73	G.E #51	Walker L15	Westinghouse # 78.	G.E #66	Westinghouse #50	G.E #55
Amperes, Standard Rating Test at 500 Volts. 75°C. rise in one hour's run	65	65	70	103	99	106	130	130	140	150	175	188	260	277.
Brake H.P developed on Standard Rating Test.	35	35	39	50	52	60	75	75	80	85	100	108	150	160
Working Maximum Current = 25% Excess Full Load, Amperes	81	81	87·5	129	124	132	162	162	175	188	219	235	325	346
Heat-rating Current = $\frac{1}{5}$ maximum = Average Current.	16	16	17·5	26	25	26	32	32	35	38	44	47	65	69
Speed-Current Relation: $\frac{A}{amperes} + y$ = Speed of Armature r.p.m. — A	20350	11050	18300	21550	11810	16400	23800	27400	23300	39600	29100	41600	47200	41200
y	281	326	310	287	300	310	314	304	469	304	341	347	378	408.
Speed at Working Maximum current, r.p.m.	532	463	520	451	395	434	461	473	602	515	474	524	523	527
Speed at Average Heat-rating current, r.p.m.	1551	1016	1356	1115	772	940	1058	1161	1135	1346	1002	1233	1105	1005

<u>EXHIBIT F.</u>

<u>Boston Motor test 100 amp. 60 % of time.</u>

Mar. 26, 1900.
Run # 13.

BOSTON ELEVATED RY. CO.

ELEVATED LINES

TEST G.E. "55" MOTOR

{ FRAME # 58853
ARM. # 66901

Average Room Temp. 22.5° C
Average Voltage 500
Average Speed 820

100 AMPERES   60% OF TIME   5 MIN. CYCLE

Current for Resistance Readings. { Armature 59 Determinations. 84 Average Amp's. 9.2 Average Seconds.
Field 59 Determinations. 62 Average Amp's. 12 Average Seconds.

Case Eng'r

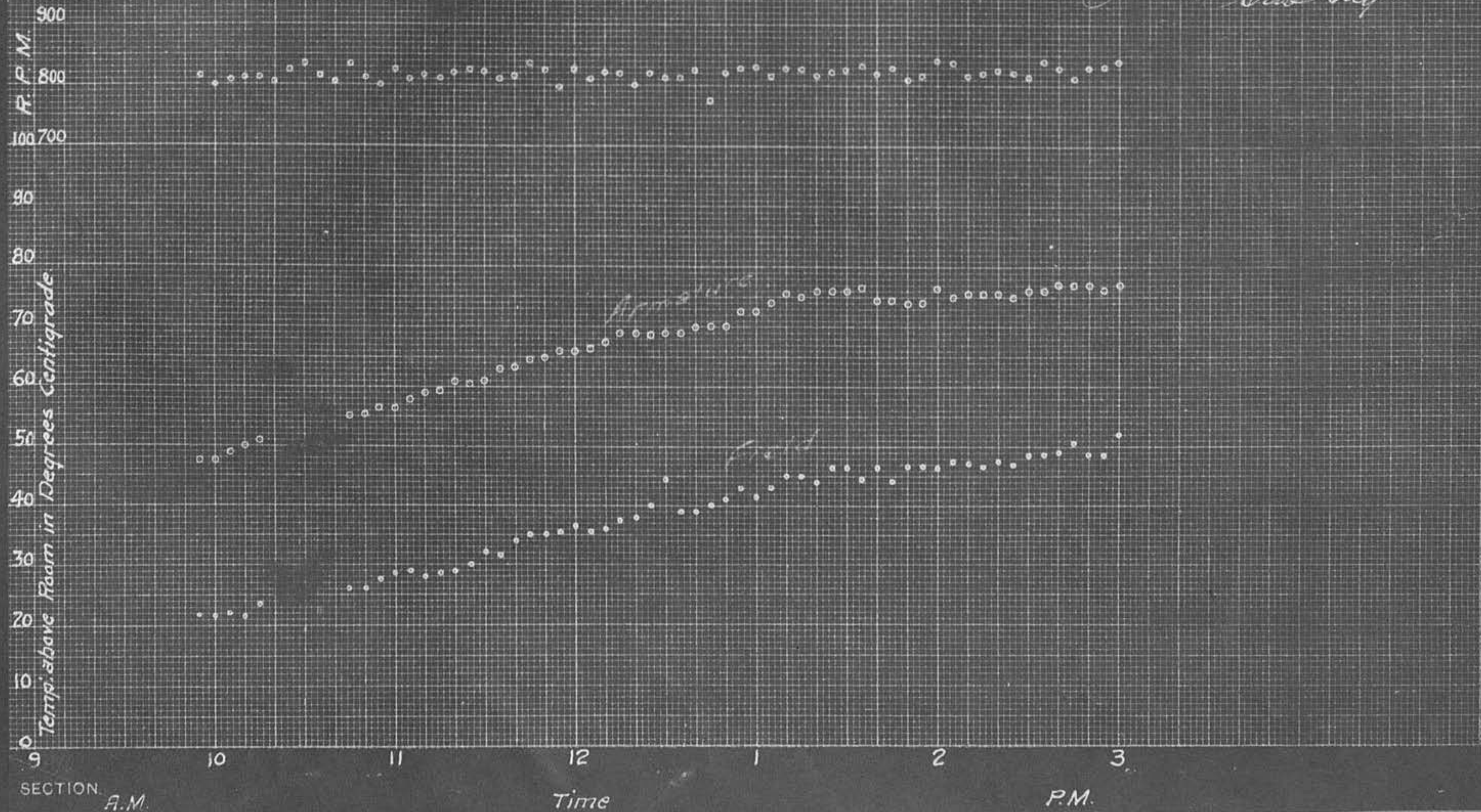


EXHIBIT G.

Boston Motor test 300 amp. 20 % of time.

BOSTON ELEVATED RY. CO.

ELEVATED LINES

Mar. 15. 1900.
Run #4

TEST G.E. "55" MOTOR

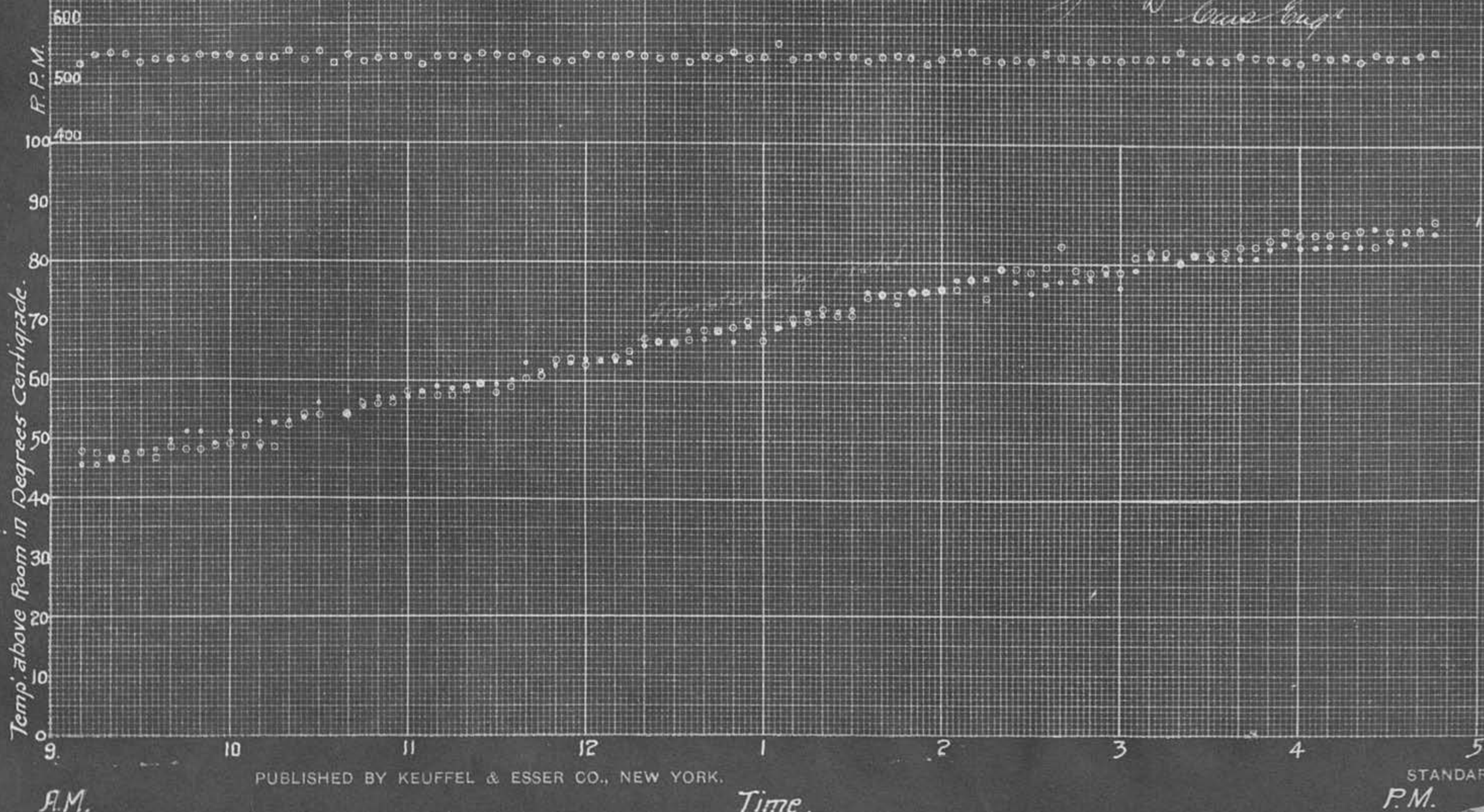
FRAME #58853
ARM. #66901

Average Room Temp. 22° C
Average Voltage 500
Average Speed. 545

300 AMPERES 20% OF TIME 5 MIN. CYCLE.

Current for Resistance Readings.

Armature	91 Determinations.		
	76 Average Amps.	6.7 Average Seconds.	
Field	91 Determinations.		
	56 Average Amps.	10.4 Average Seconds.	



PUBLISHED BY KEUFFEL & ESSER CO., NEW YORK.

STANDARD

EXHIBIT H.

Train movement and current curves.

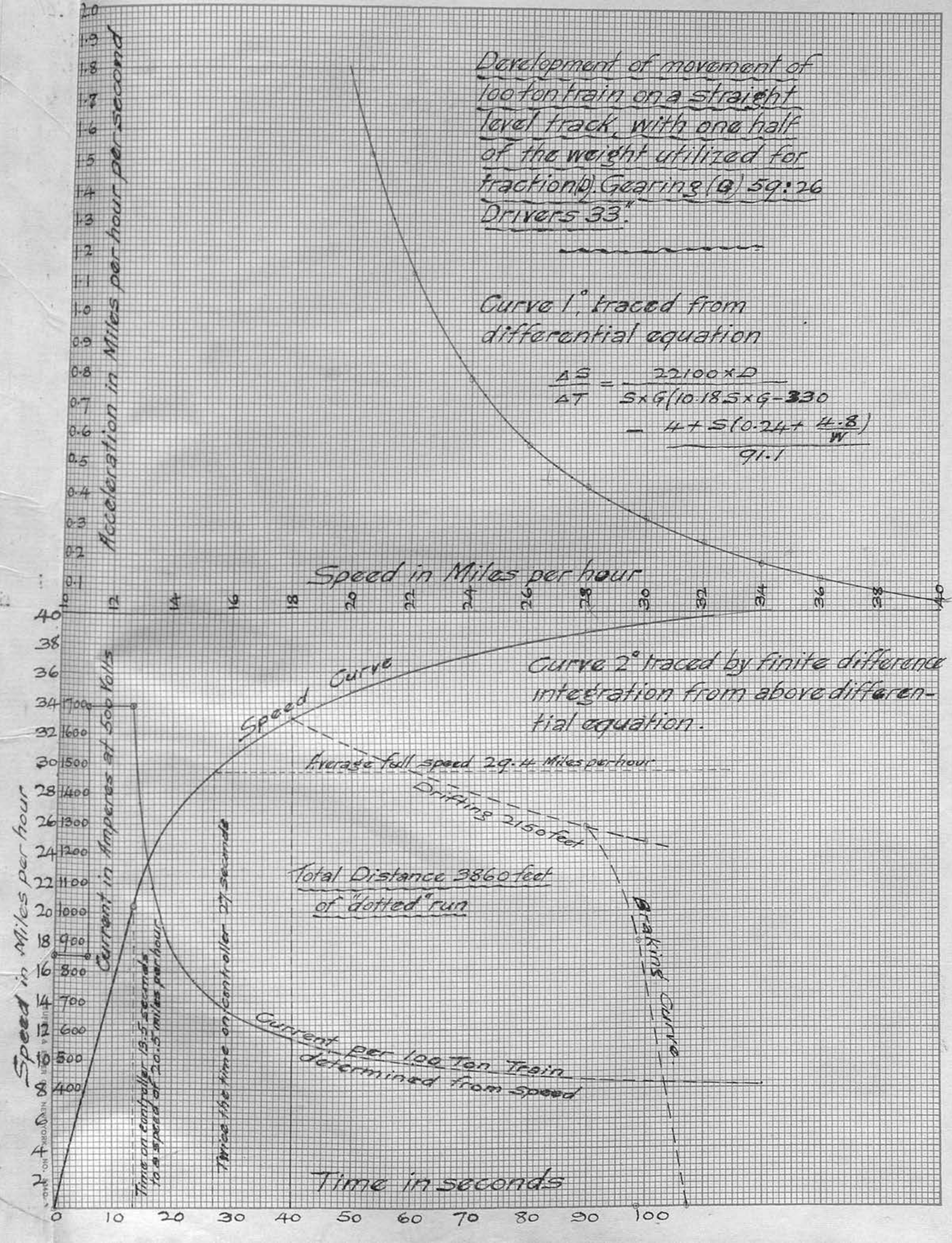

Development of movement of 100 ton train on a straight level track, with one half of the weight utilized for traction(D). Gearing (G) 59:26 Drivers 33".
Curve 1°, traced from differential equation
ΔS/ΔT = 22100×D / S×G(10.18S×G−330 − 4+S(0.24+4.8/W) / 91.1
Acceleration in Miles per hour per second
Speed in Miles per hour
Curve 2° traced by finite difference integration from above differential equation.
Speed Curve
Average full speed 29.4 Miles per hour
Drifting 2150 feet
Total Distance 3860 feet of "dotted" run
Braking Curve
Current in Amperes at 500 Volts
Time on controller 13.5 seconds to a speed of 20.5 miles per hour
Twice the time on controller 27 seconds
Current per 100 Ton Train determined from speed
Speed in Miles per hour
Time in seconds

EXHIBIT I.

Train resistance record. 21 ton train.

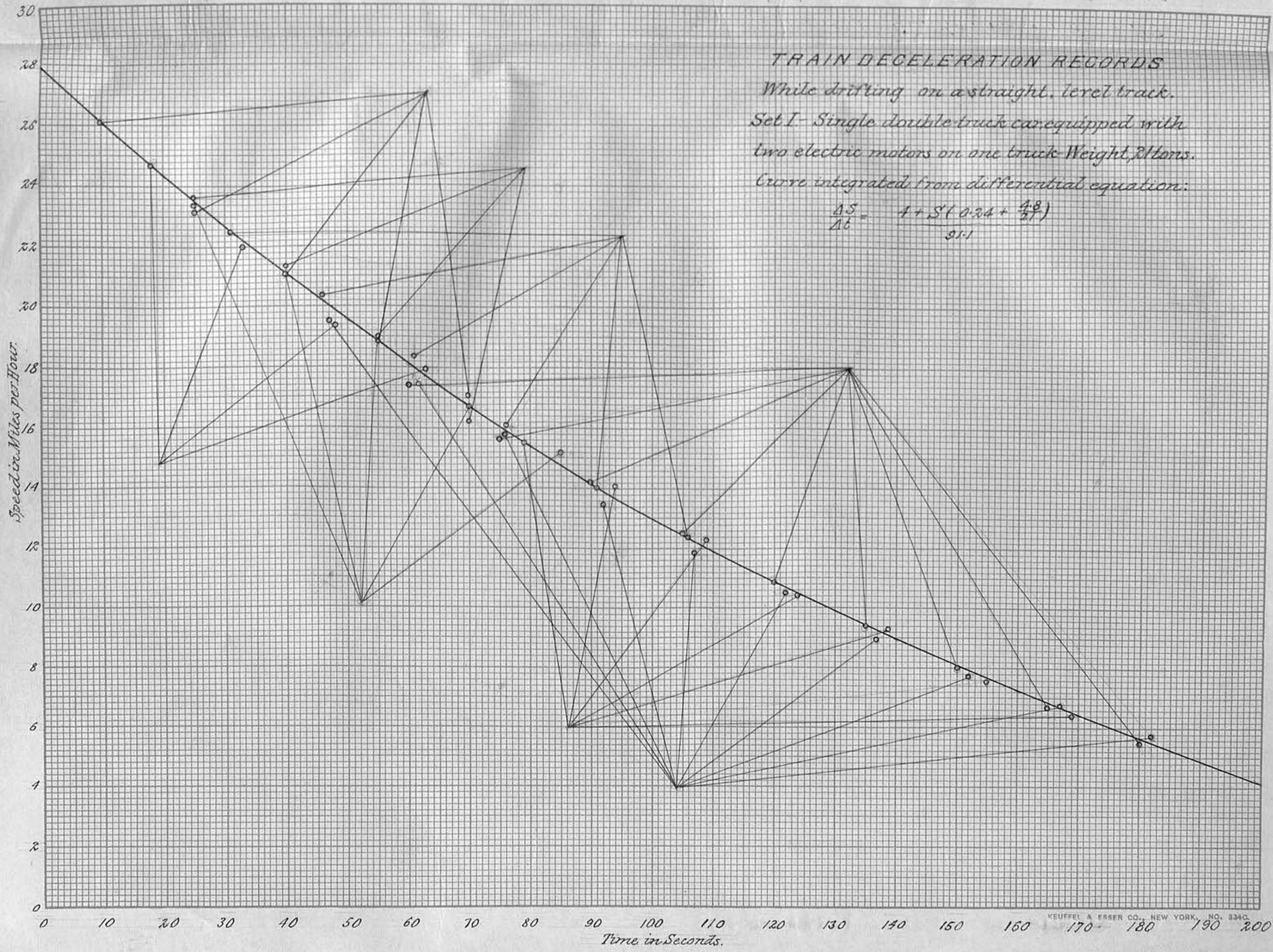
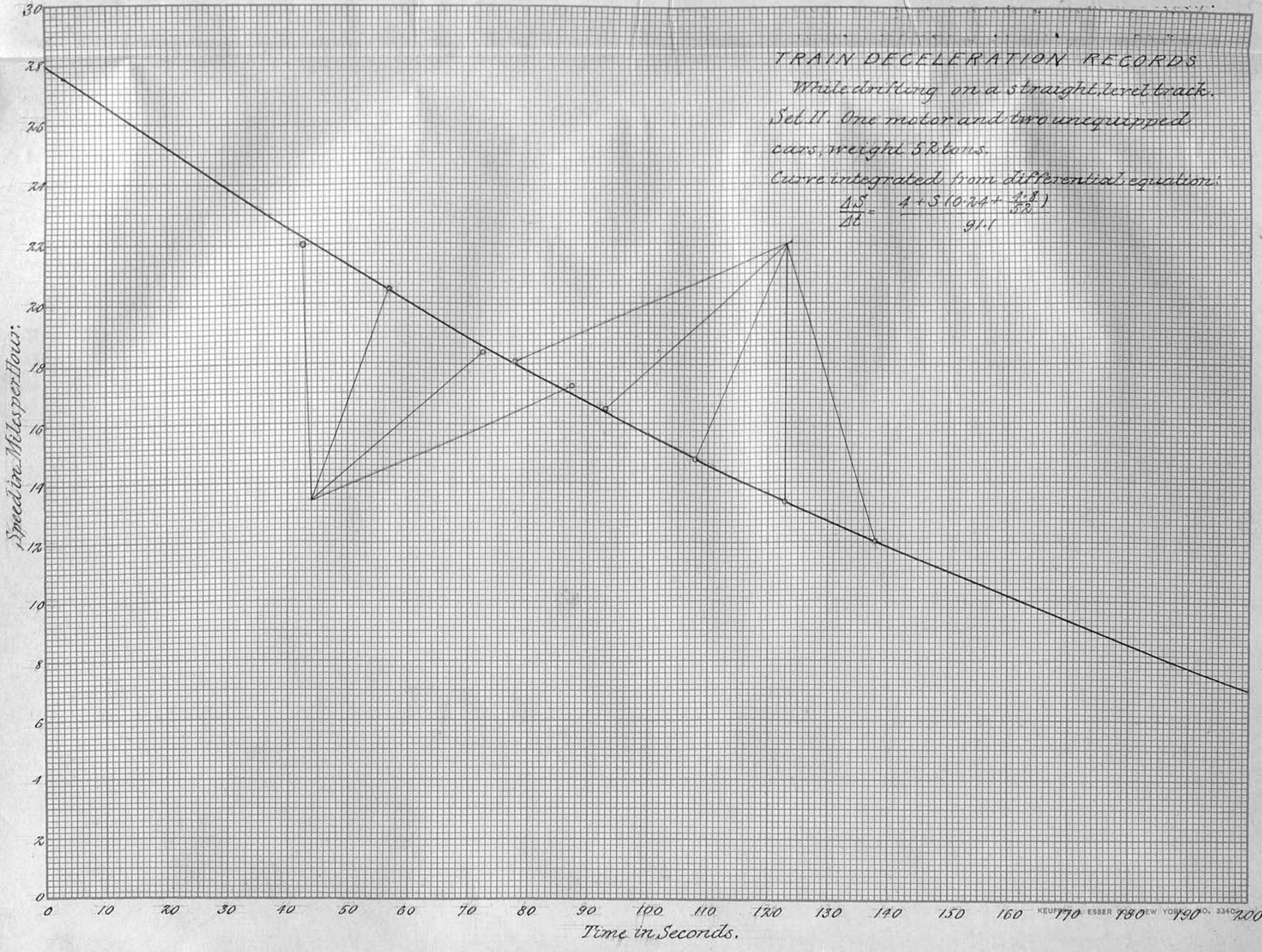

TRAIN DECELERATION RECORDS
While drifting on a straight, level track.
Set I - Single double-truck car equipped with two electric motors on one truck. Weight, 21 tons.
Curve integrated from differential equation:
$\frac{\Delta S}{\Delta t} = \frac{4 + S\left(0.24 + \frac{4.8}{21}\right)}{91.1}$
Speed in Miles per Hour.
Time in Seconds.
0 2 4 6 8 10 12 14 16 18 20 22 24 26 28 30
0 10 20 30 40 50 60 70 80 90 100 110 120 130 140 150 160 170 180 190 200
KEUFFEL & ESSER CO., NEW YORK, NO. 334C.

EXHIBIT J.

Train resistance record. 52 ton train.



TRAIN DECELERATION RECORDS
While drifting on a straight, level track.
Set II. One motor and two unequipped cars, weight 52 tons.
Curve integrated from differential equation:
$$\frac{\Delta S}{\Delta t} = \frac{4 + S\left(0.24 + \frac{4.8}{52}\right)}{91.1}$$
Speed in Miles per Hour.
Time in Seconds.
0
2
4
6
8
10
12
14
16
18
20
22
24
26
28
30
0
10
20
30
40
50
60
70
80
90
100
110
120
130
140
150
160
170
180
190
200

EXHIBIT K.

Train resistance record. 62 ton train.

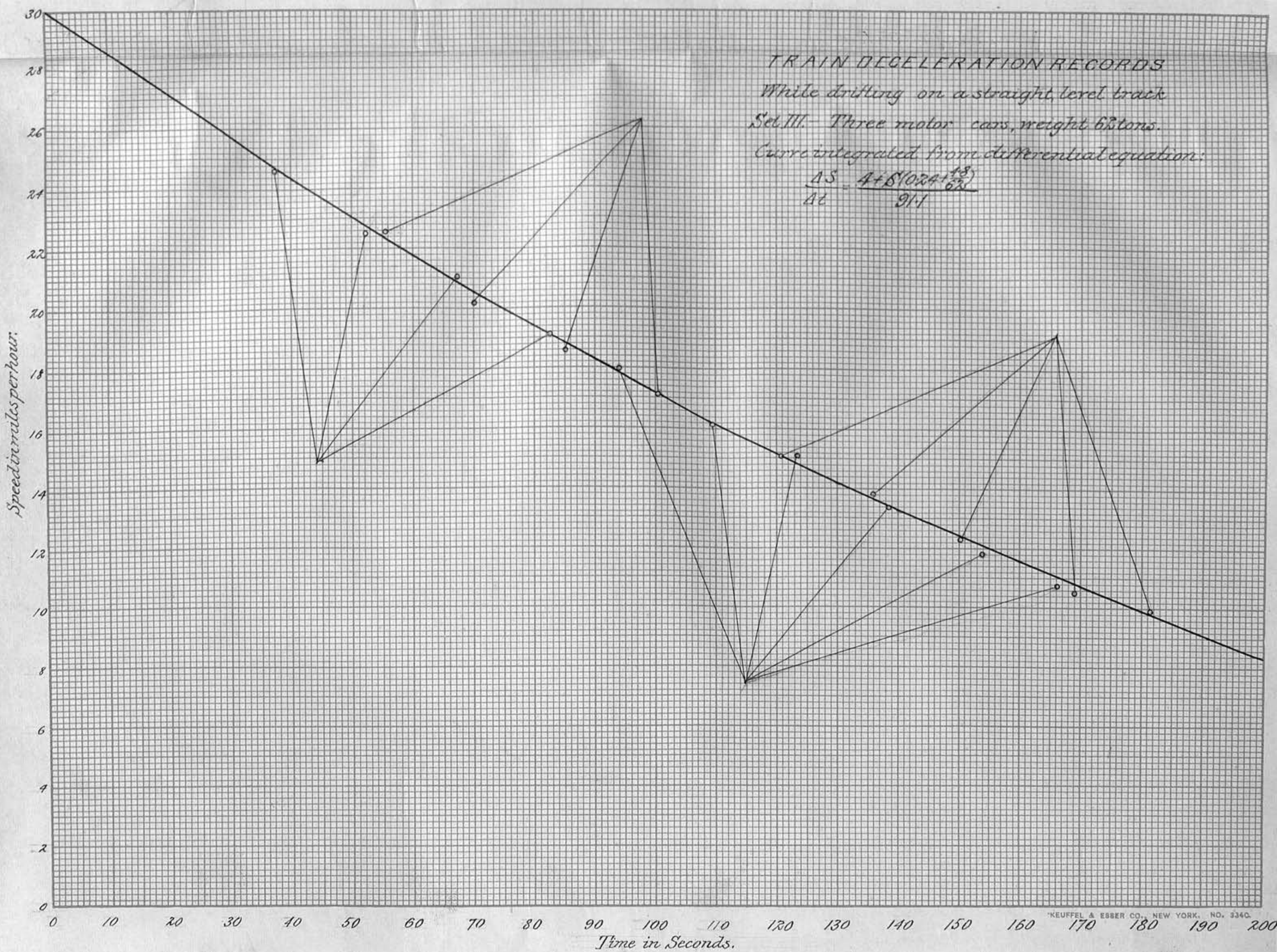
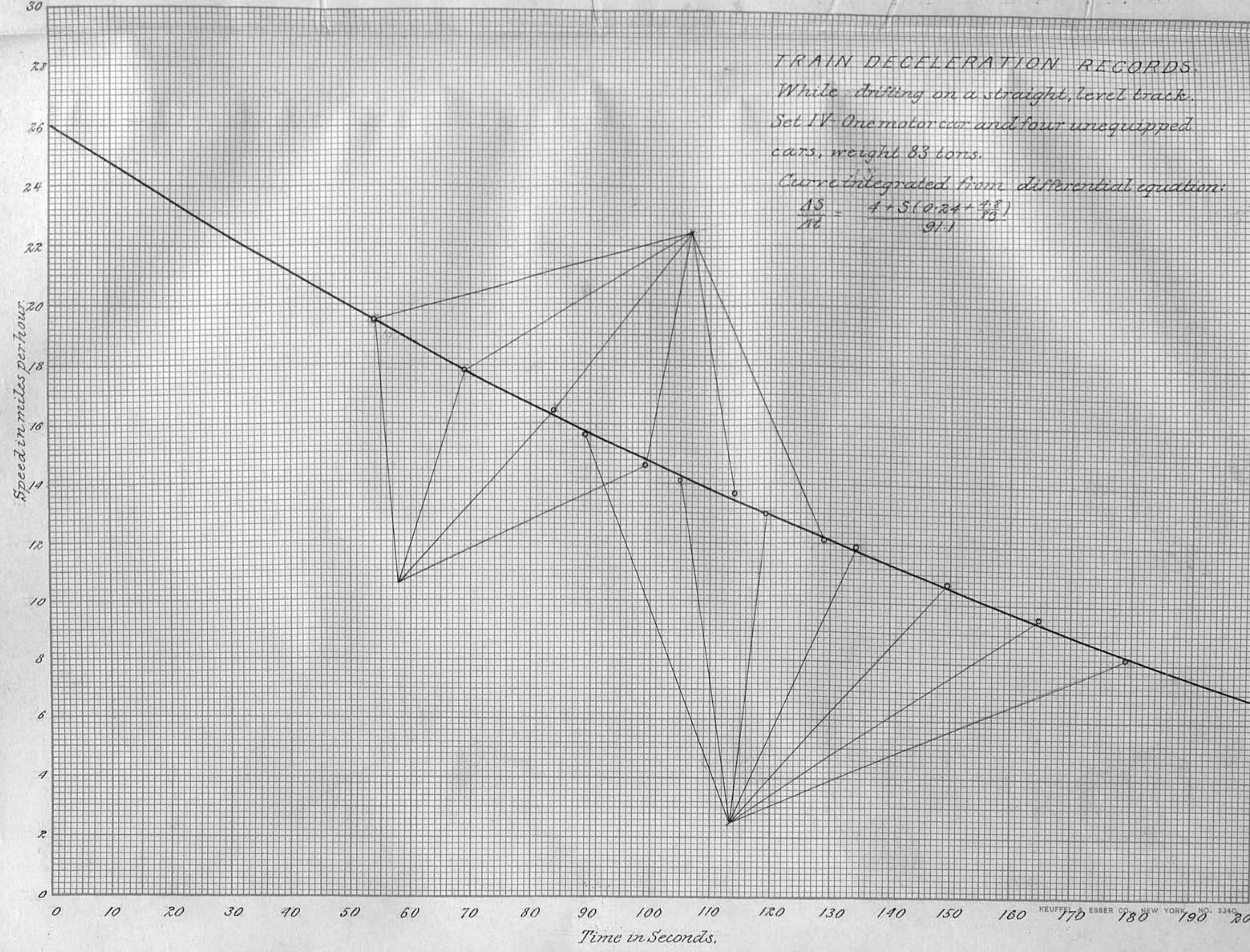

TRAIN DECELERATION RECORDS
While drifting on a straight level track
Set III. Three motor cars, weight 62 tons.
Curve integrated from differential equation:
$\frac{\Delta S}{\Delta t} = \frac{4 + S\left(0.24 + \frac{48}{62}\right)}{91.1}$
Speed in miles per hour.
Time in Seconds.
0
2
4
6
8
10
12
14
16
18
20
22
24
26
28
30
0
10
20
30
40
50
60
70
80
90
100
110
120
130
140
150
160
170
180
190
200
KEUFFEL & ESSER CO., NEW YORK. NO. 3340.

EXHIBIT L.

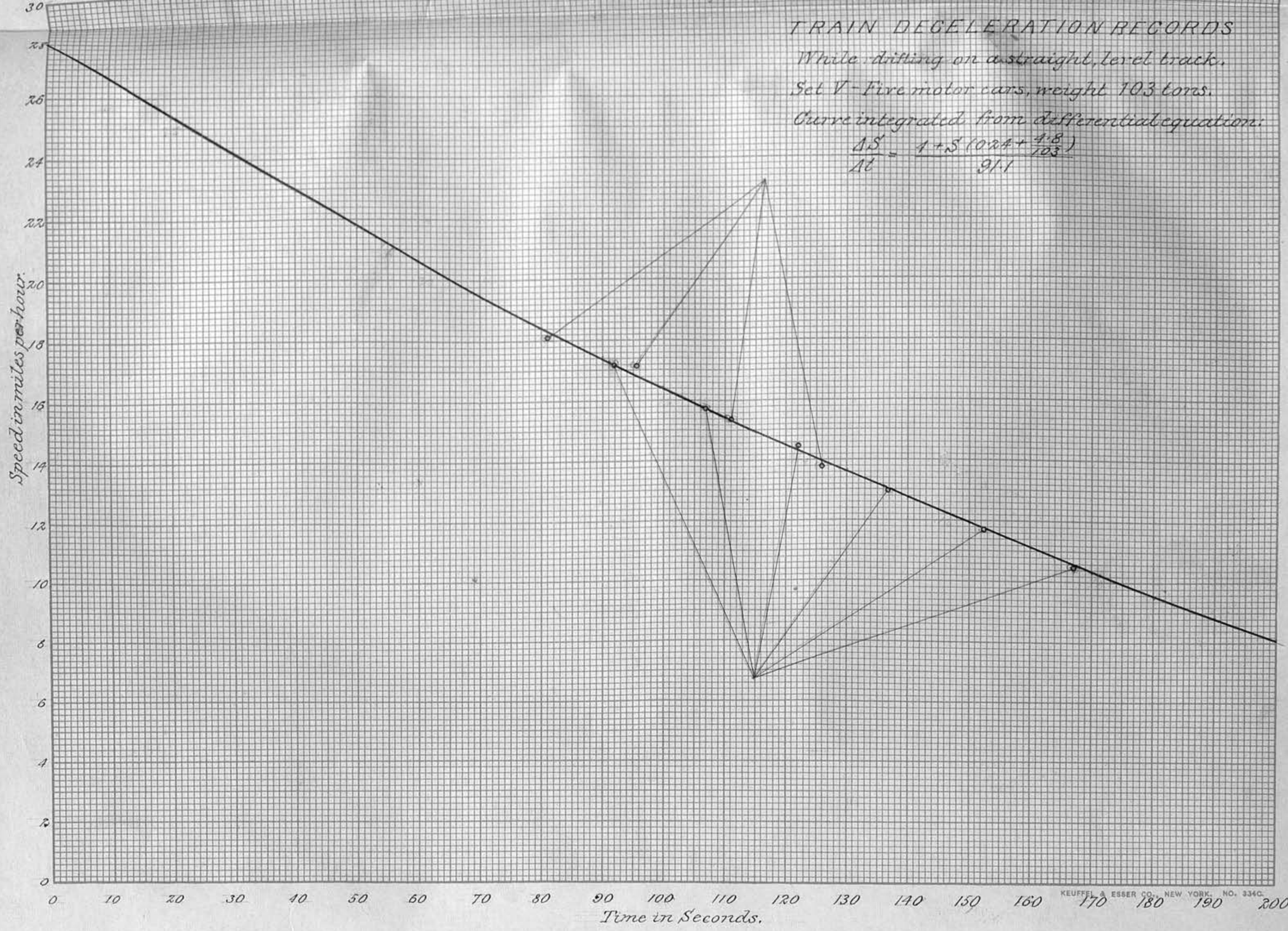
Train resistance record. 83 ton train.



TRAIN DECELERATION RECORDS.
While drifting on a straight, level track.
Set IV - One motor car and four unequipped cars, weight 83 tons.
Curve integrated from differential equation:
$\frac{\Delta S}{\Delta t} = \frac{4 + S(0.24 + \frac{4.8}{83})}{91.1}$
Speed in miles per hour.
Time in Seconds.
0
2
4
6
8
10
12
14
16
18
20
22
24
26
28
30
0
10
20
30
40
50
60
70
80
90
100
110
120
130
140
150
160
170
180
190
200
KEUFFEL & ESSER CO. NEW YORK. NO. 3340

EXHIBIT M.

Train resistance record. 103 ton train.



TRAIN DECELERATION RECORDS
While drifting on a straight, level track.
Set V - Five motor cars, weight 103 tons.
Curve integrated from differential equation:
$\frac{\Delta S}{\Delta t} = \frac{1 + S\left(0.24 + \frac{4.8}{103}\right)}{91.1}$
Speed in miles per hour.
Time in Seconds.
0
10
20
30
40
50
60
70
80
90
100
110
120
130
140
150
160
170
180
190
200
0
2
4
6
8
10
12
14
16
18
20
22
24
26
28
30
KEUFFEL & ESSER CO., NEW YORK. NO. 334C.

EXHIBIT N.

Graphic determination of the economic Motor equipment for given conditions.

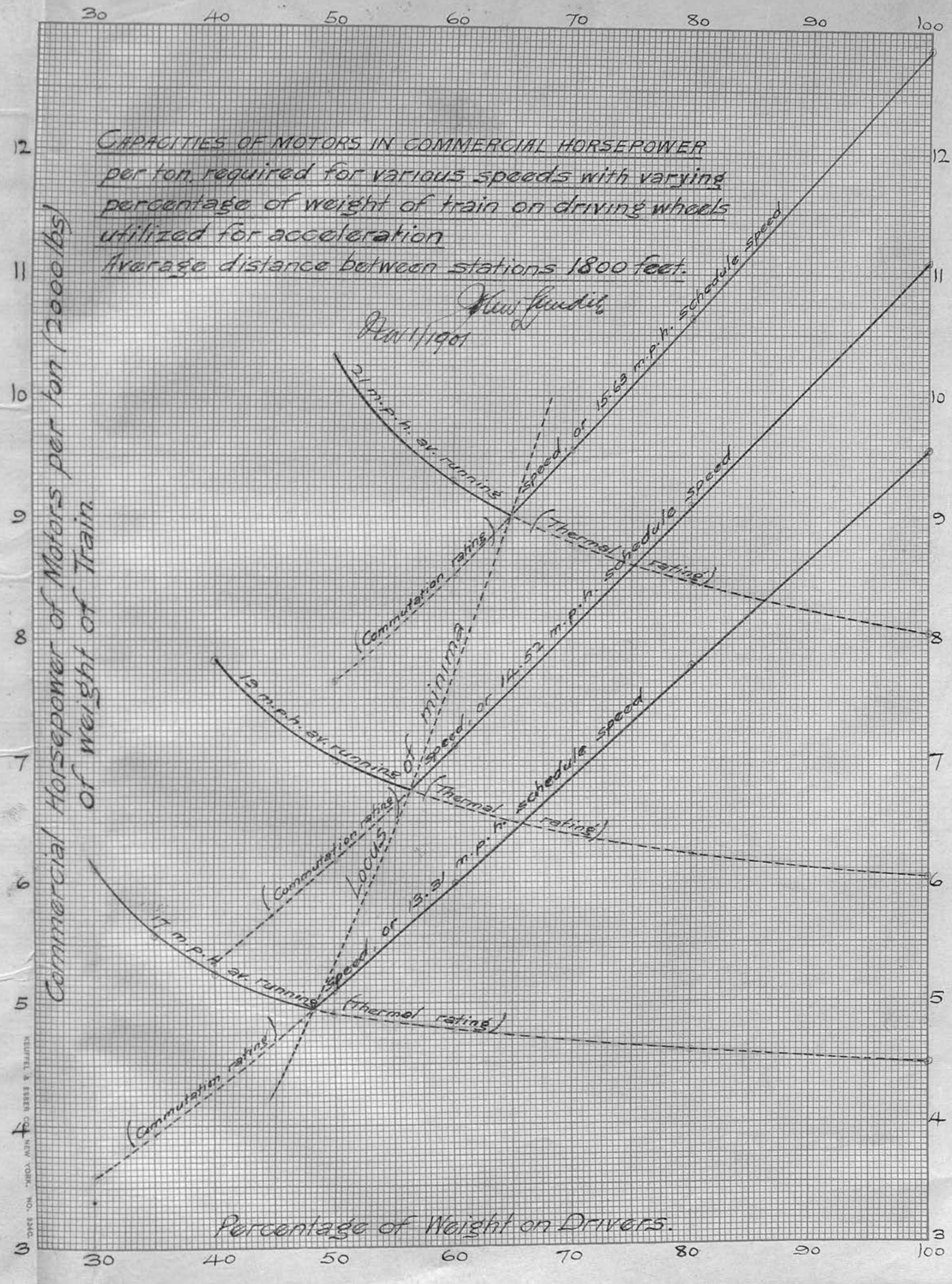
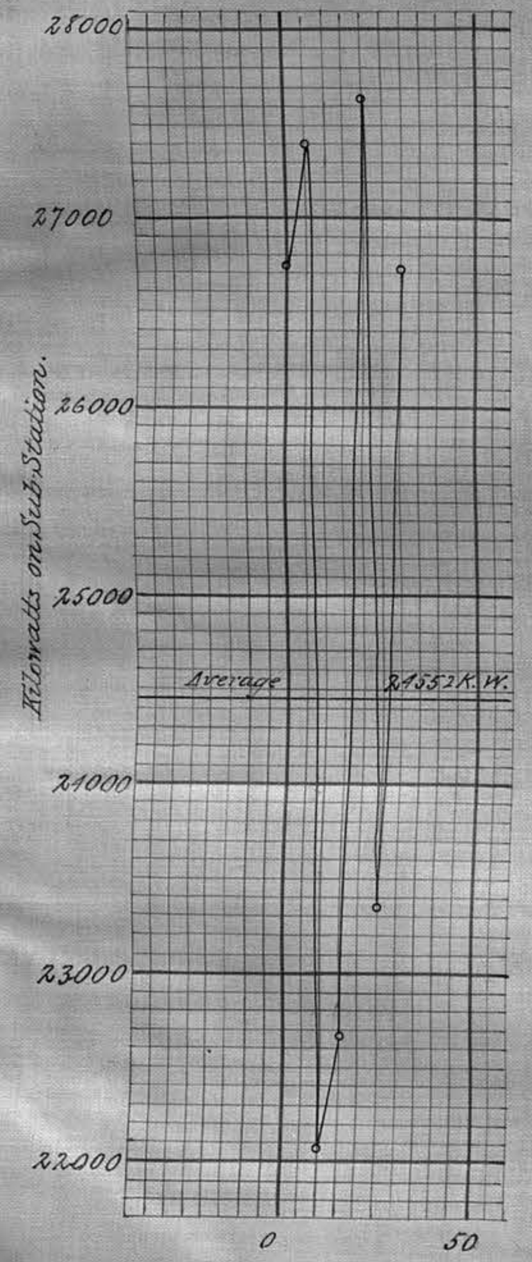
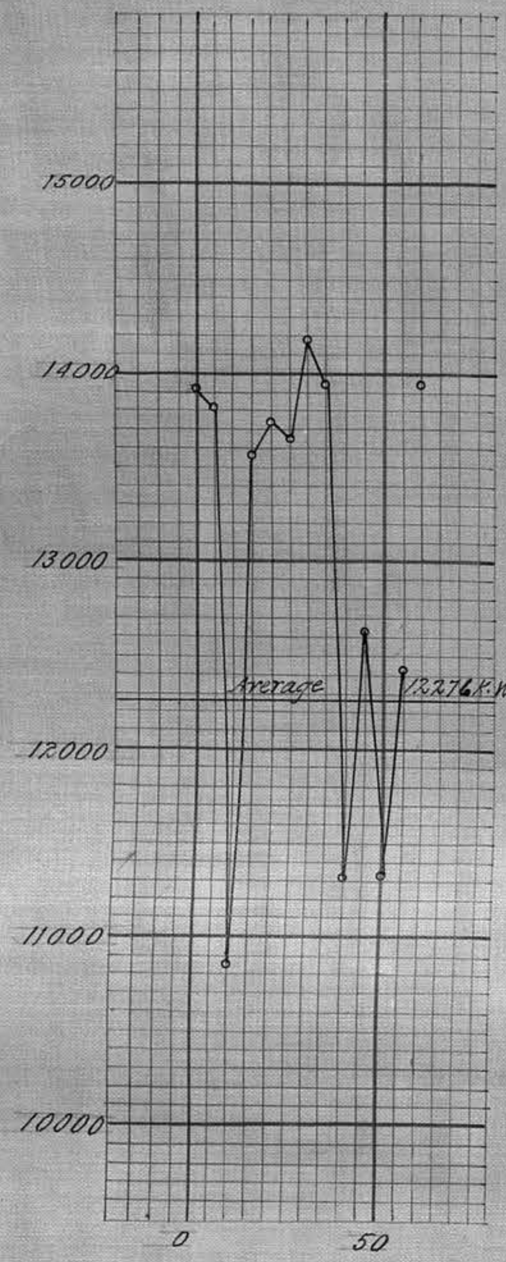

CAPACITIES OF MOTORS IN COMMERCIAL HORSEPOWER
per ton required for various speeds with varying
percentage of weight of train on driving wheels
utilized for acceleration
Average distance between stations 1800 feet.
21 m.p.h. av. running speed, or 15.63 m.p.h. schedule speed
19 m.p.h. av. running speed, or 14.52 m.p.h. schedule speed
17 m.p.h. av. running speed, or 13.31 m.p.h. schedule speed
(Thermal rating)
(Thermal rating)
(Thermal rating)
(Commutation rating)
(Commutation rating)
(Commutation rating)
Locus of minima
Commercial Horsepower of Motors per ton (2000 lbs) of weight of Train.
Percentage of Weight on Drivers.
30
40
50
60
70
80
90
100
3
4
5
6
7
8
9
10
11
12

EXHIBIT O.

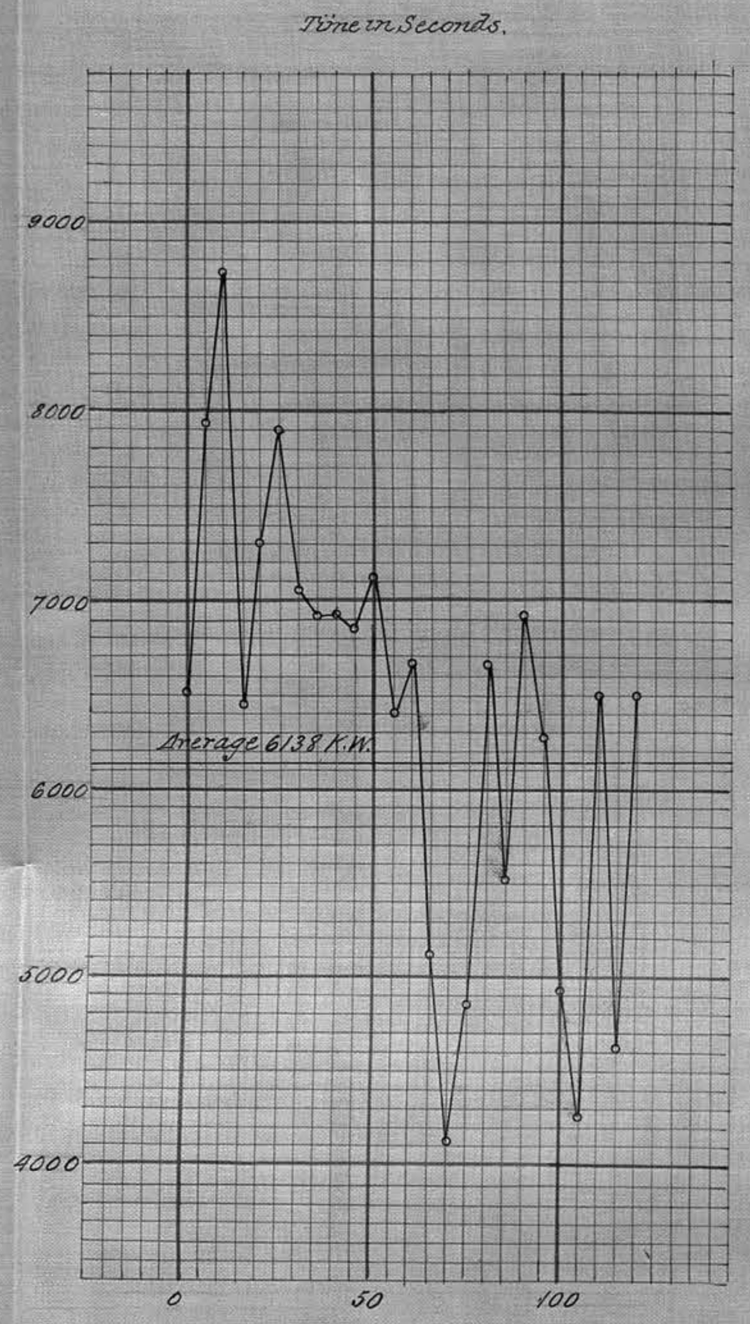
Fluctuation diagram for estimated Manhattan Railway conditions.



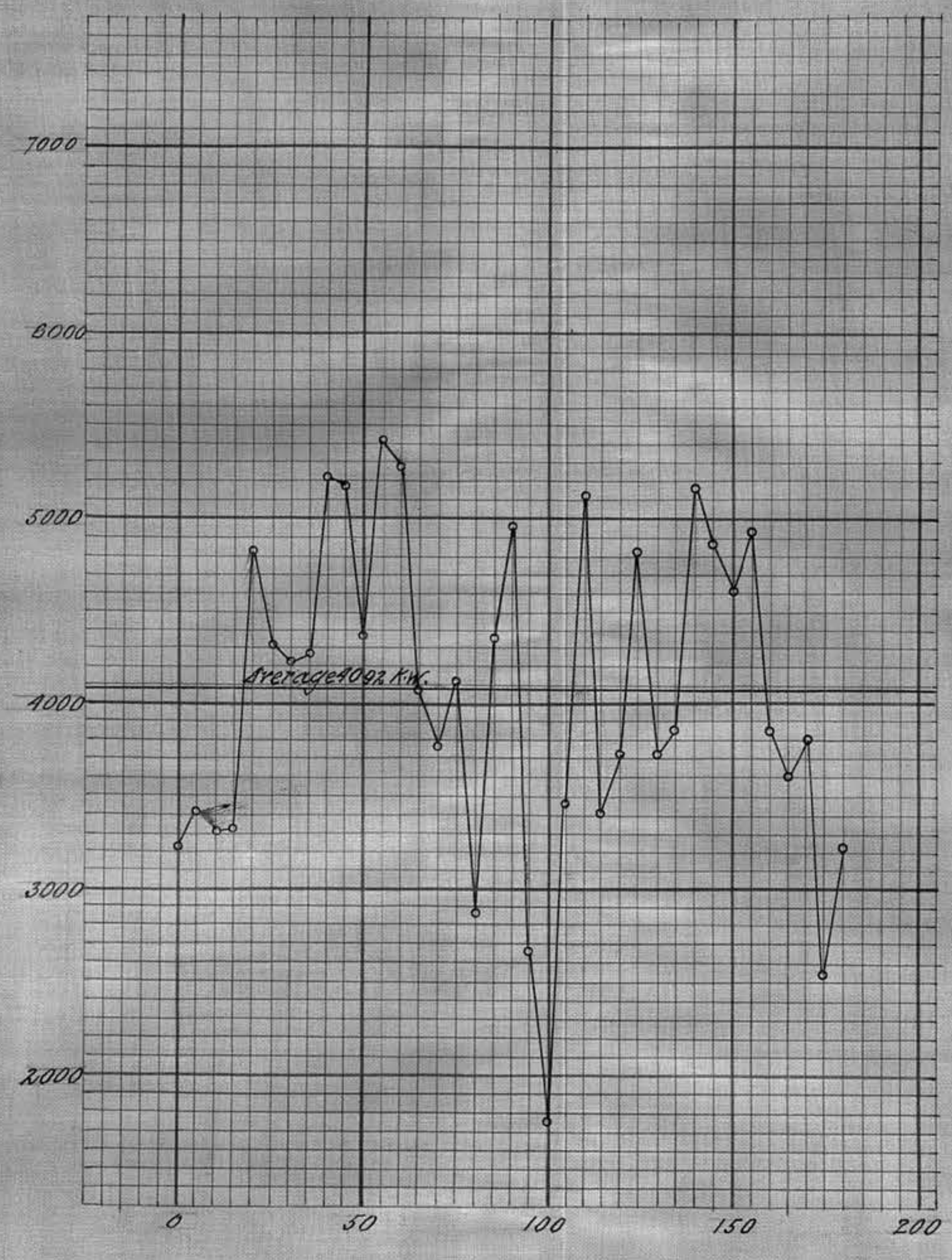
Train Interval 30 Seconds
88 Trains in Circuit.



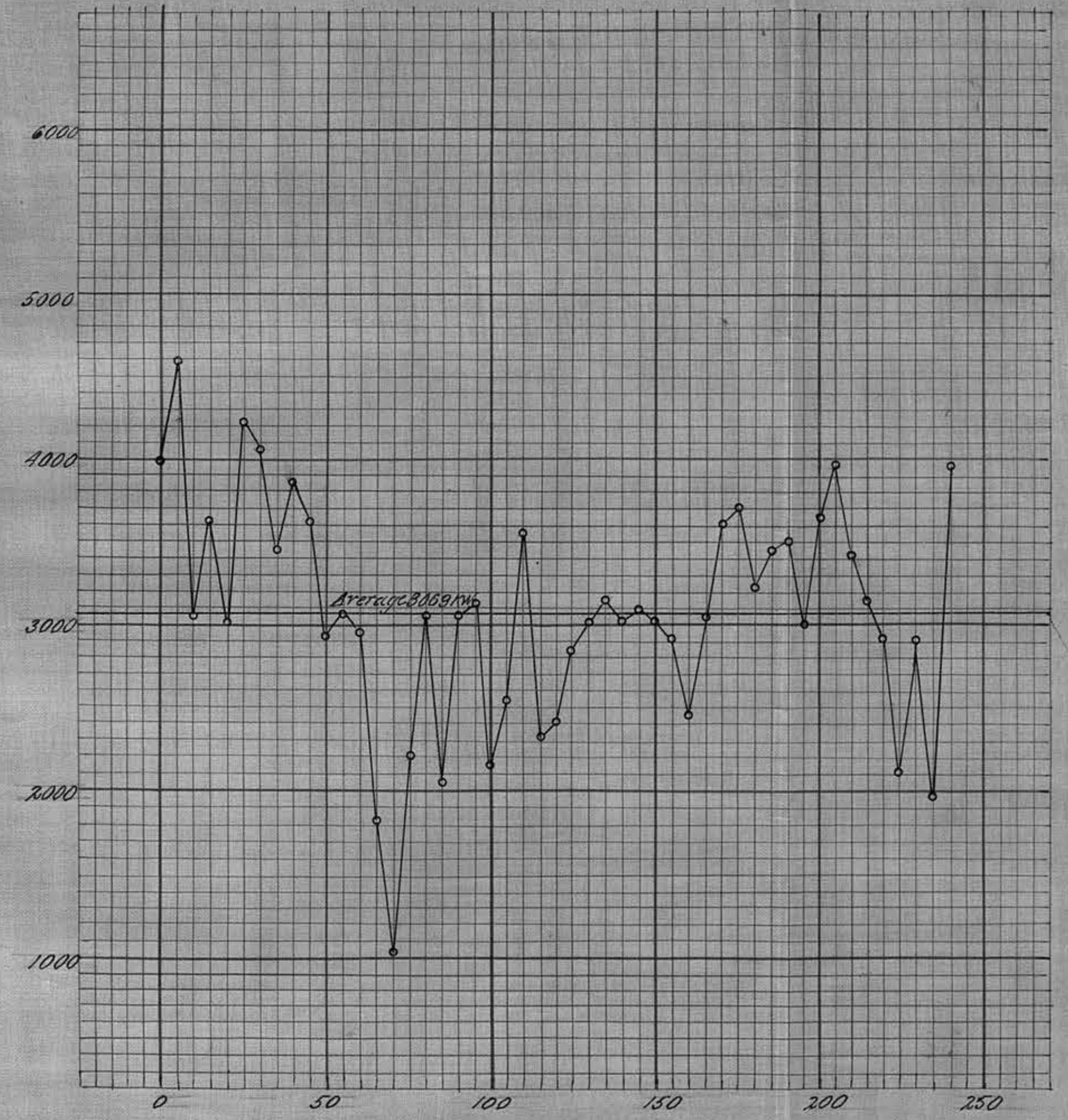
Train Interval 1 Minute.
44 Trains in Circuit.



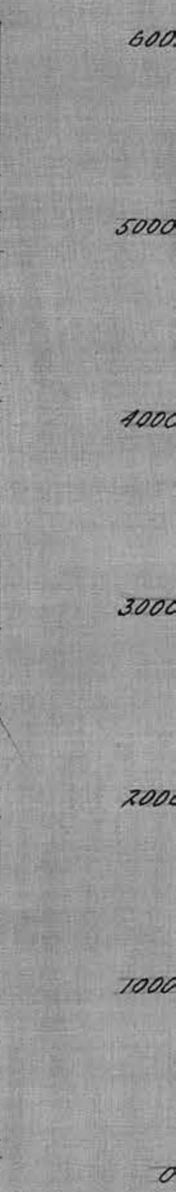
Train Interval 2 Minutes.
22 Trains in Circuit.



Train Interval 3 Minutes
14.67 Trains in Circuit.

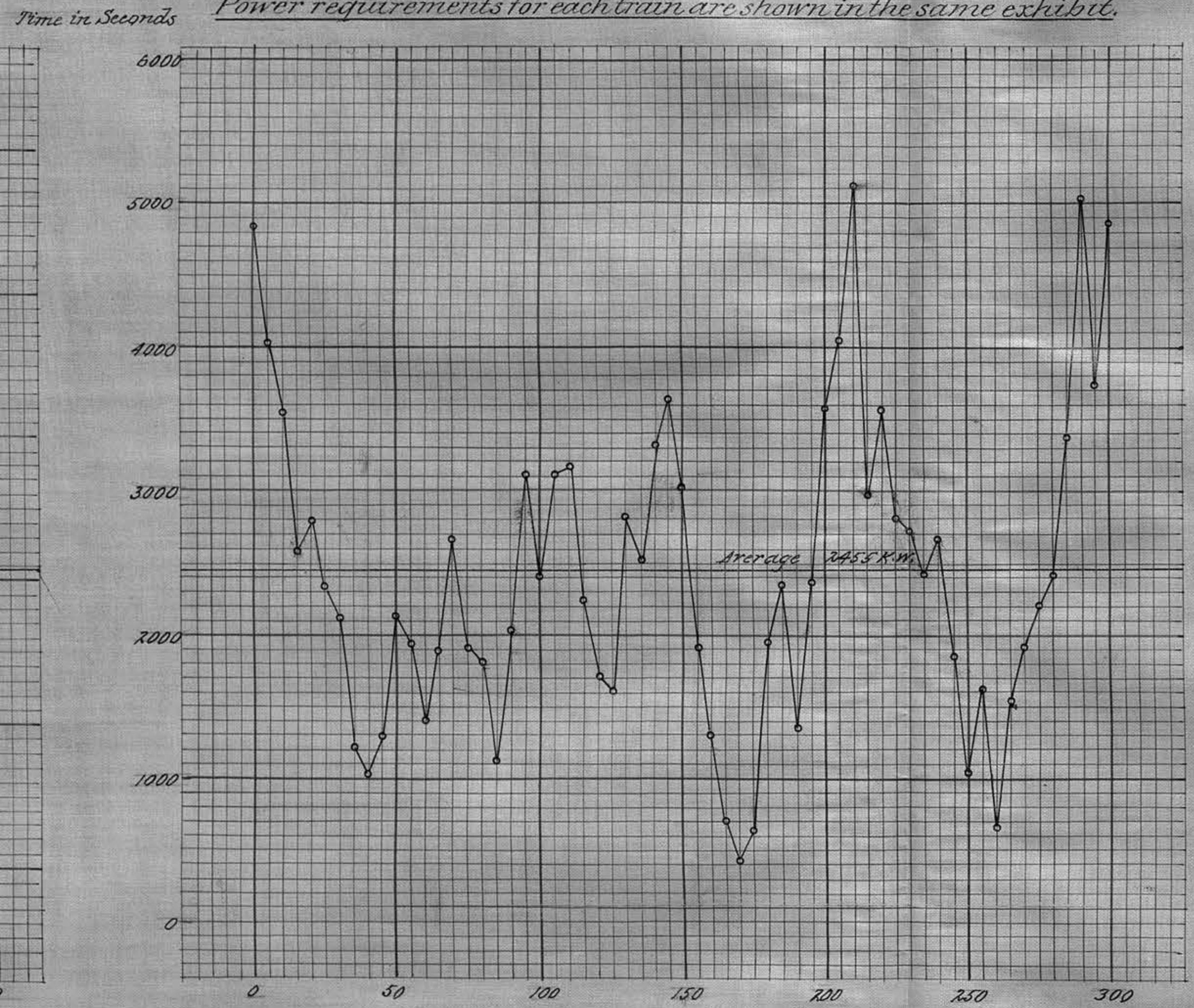


Train Interval 4 Minutes.
11 Trains in Circuit.



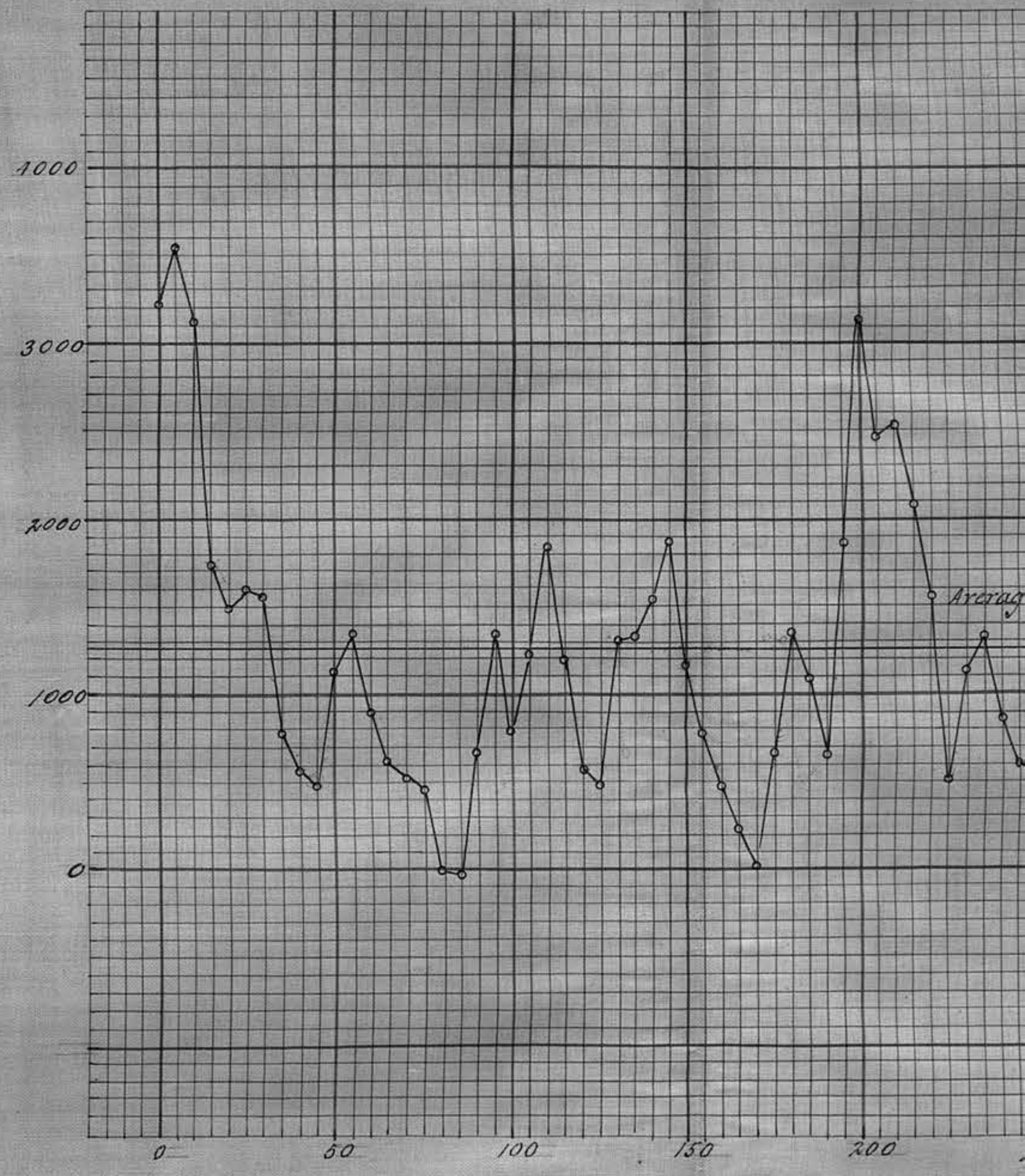
ESTIMATED FLUCTUATIONS OF POWER

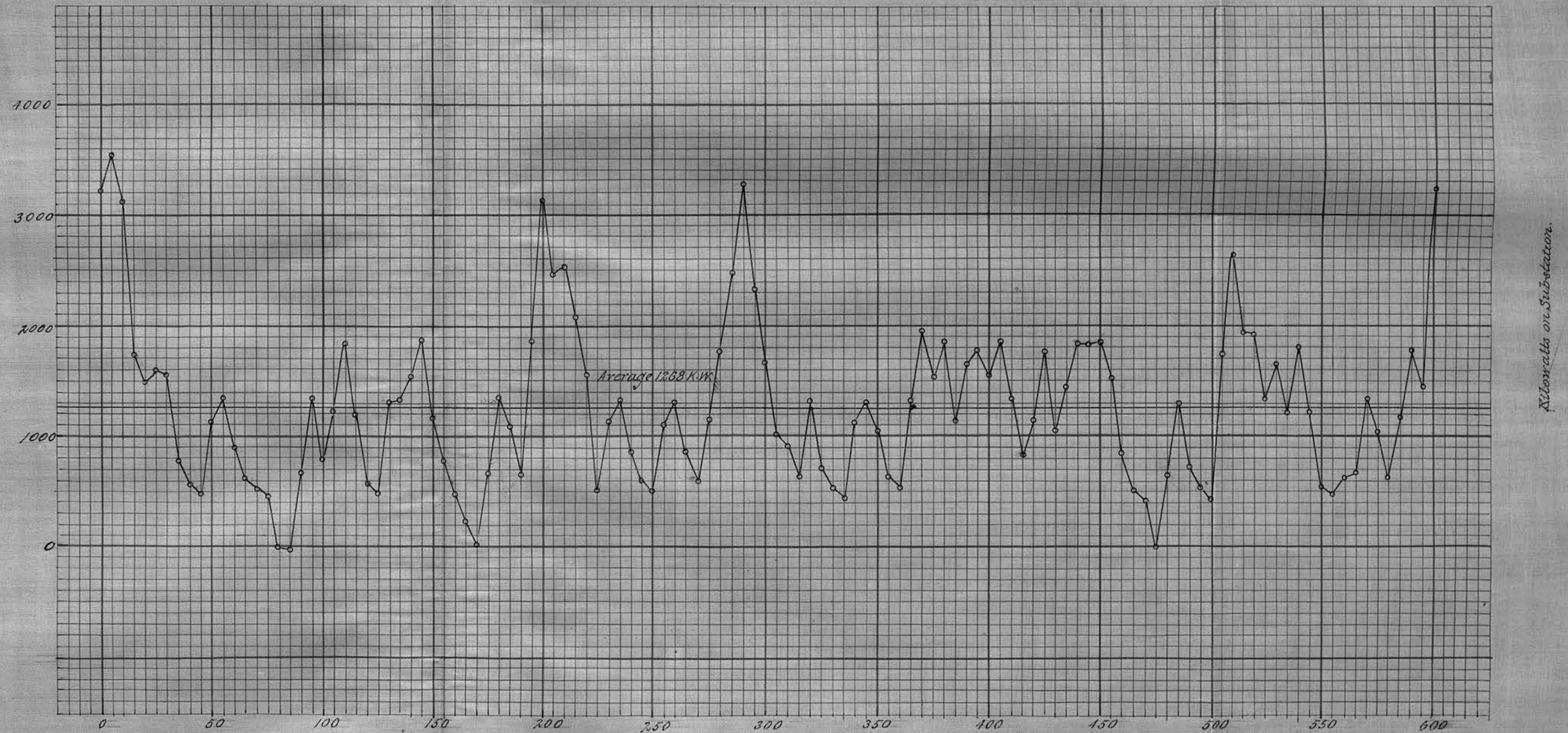
On Substation feeding that section of Manhattan Elevated Railroad shown in Exhibit P for varying numbers of trains in circuit. Power requirements for each train are shown in the same exhibit.



Train Interval 5 Minutes.
8.80 Trains in Circuit.

Train Interval 6 Minutes.
7.33 Trains in Circuit.





Train Interval 10 Minutes.
4.40 Trains in Circuit.

EXHIBIT Q.

Recorded fluctuations on generating station of South Side Elevated Railway, Chicago.

South Side Elevated Railway Company

Chicago, Illinois.

Record of Fluctuations on Generating Station

Between 5:30 and 6:30 p.m.

September 26, 1898.

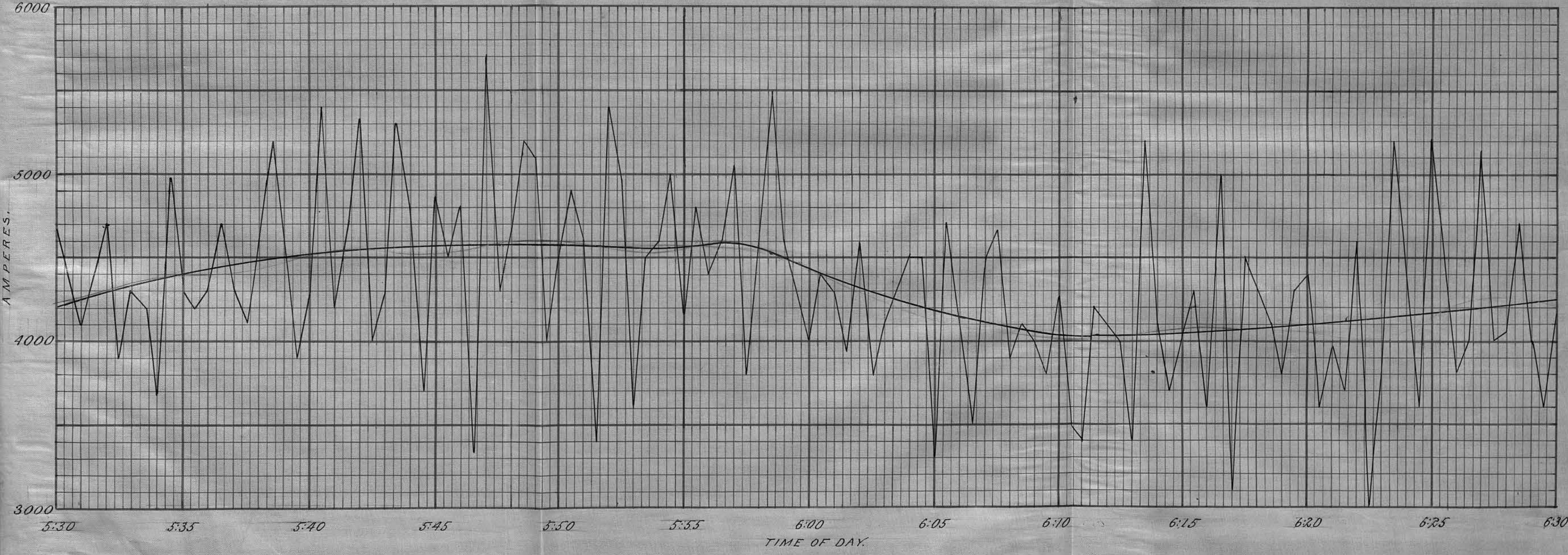


EXHIBIT R.

Recorded fluctuations on 12th Street Battery Substation

of South Side Elevated Railway, Chicago.

SOUTH SIDE ELEVATED RAILWAY COMPANY

Chicago, Illinois.

RECORD OF FLUCTUATIONS ON TWELFTH STREET BATTERY STATION

Between 5:30 and 6:30 p.m.

September 20, 1898.

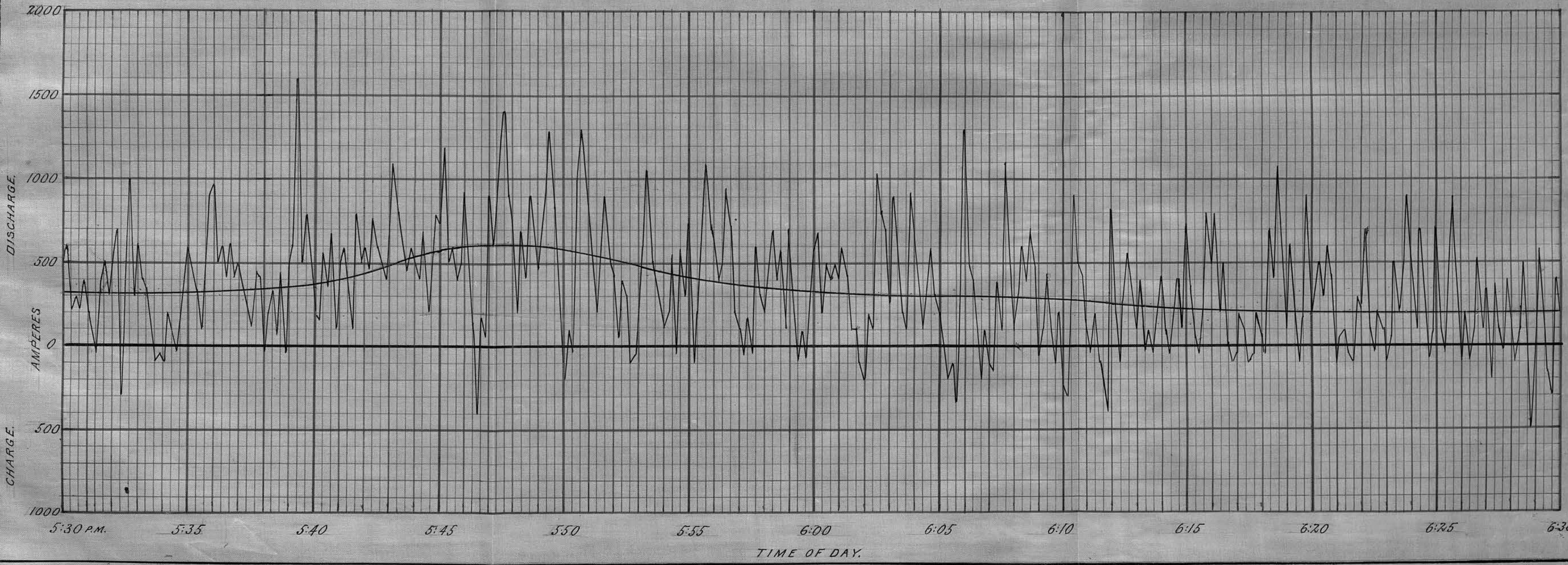


EXHIBIT S.

Recorded fluctuations on 61st Street Battery Substation of the South Side Elevated Railway, Chicago.

SOUTH SIDE ELEVATED RAILWAY COMPANY

Chicago, Illinois.

RECORD OF FLUCTUATIONS ON SIXTY FIRST STREET BATTERY STATION

Between 5:30 and 6:30 p.m.

September 27, 1898.